P[actual = "blue"] = 0.2
P[actual = "yellow"] = 0.8

P[report = "blue" | actual = "blue"] = 0.8
P[report = "yellow" | actual = "blue"] = 0.2

P[report = "yellow" | actual = "yellow"] = 0.8
P[report = "blue" | actual = "yellow"] = 0.2

P[actual = "yellow" | report = "yellow"] = P[actual = "yellow"] * P[report = "yellow" | actual = "yellow"]

P[report = "yellow"] = P[actual = "yellow"] * P[report = "yellow" | actual = "yellow"] + P[actual = "blue"] * P[report = "yellow" | actual = "blue"]

= 0.2*0.8 + 0.8*0.2 = 0.32

P[actual = "yellow" | report = "yellow"] = 0.2*0.8 / 0.32 = 0.5

--

P[emails are spam] = 0.4
P[emails are not spam] = 0.6

P[word = "checkout" | email is spam] = 0.01
P[word = "checkout" | email is NOT spam] = 0.004

P[email is spam | word = "checkout"] = P[email is spam] * P[word = "checkout" | email is spam] / P[word = "checkout"]

P[word = "checkout"] = P[email is spam] * P[word = "checkout" | email is spam] +
P[\text{email is not spam}] \times P[\text{word = "checkout" | email is NOT spam}] = 0.4 \times 0.01 + 0.6 \times 0.004 = 0.0064

P[\text{email is spam | word = "checkout"}] = 0.4 \times 0.01 / 0.0064 = 0.625

\begin{align*}
P[\text{majority-blue}] &= 1/2 \\
P[\text{majority-red}] &= 1/2 \\

P[\text{ball picked is "blue" | urn is "majority-blue"}] &= 2/3 \\
P[\text{ball picked is "red" | urn is "majority-blue"}] &= 1/3 \\

P[\text{ball picked is "red" | urn is "majority-red"}] &= 2/3 \\
P[\text{ball picked is "blue" | urn is "majority-red"}] &= 1/3 \\

P[\text{urn is majority-blue | ball picked is blue}] &= P[\text{urn is majority-blue}] \times P[\text{ball picked is blue | urn is majority blue}] / P[\text{ball picked is blue}] \\

P[\text{ball picked is blue}] &= P[\text{urn is majority-blue}] \times P[\text{ball picked is "blue" | urn is "majority-blue"}] + P[\text{urn is majority-red}] \times P[\text{ball picked is "blue" | urn is "majority-red"}] \\
&= 1/2 \times 2/3 + 1/2 \times 1/3 = 1/2 \\
P[\text{urn is majority-blue | ball picked is blue}] &= 1/2 \times 2/3 / 0.5 = 2/3

\end{align*}
CSC434-Fall2014-information-cascade

majority-blue] * P[ball picked is red | urn is majority-blue] = 2/3*2/3*1/3 = 4/27

P[balls picked are blue, blue, red | urn is majority-red] = 1/3*1/3*2/3 = 2/27

P[balls picked are blue, blue, red] = 1/2*4/27 + 1/2*2/27 = 1/2*6/27 = 1/9

2/27 / 1/9 = 2/3

-----------------------------

P[see "blue" | hypothesis is "majority-blue"] - high signal (q) > 1/2
P[see "red" | hypothesis is "majority-blue"] - low signal (1-q)

P[see "red" | hypothesis is "majority-red"] - low signal (q) > 1/2
P[see "blue" | hypothesis is "majority-red"] - high signal (1-q)

S = sequence of 'a' high signals and sequence of 'b' low signals


= p*q^a *(1-q)^b + (1-p)*(1-q)^a * q^b

P[majority-blue | S] = p*q^a *(1-q)^b

-------------------------------------------------------

p*q^a *(1-q)^b + (1-p)*(1-q)^a *

q^b

a > b

Since q > 1/2
\[
q^a \ast (1-q)^b > (1-q)^a \ast q^b
\]

\[
p \ast q^a \ast (1-q)^b + (1-p) \ast (1-q)^a \ast q^b < p \ast q^a \ast (1-q)^b + (1-p) \ast q^a \ast (1-q)^b
\]

\[
1/() > 1/()
\]

\[
P[\text{majority-blue | } S] > p
\]

--

\[
a < b
\]

\[
P[\text{majority-blue | } S] = \frac{p \ast q^a \ast (1-q)^b}{p \ast q^a \ast (1-q)^b + (1-p) \ast (1-q)^a \ast q^b}
\]

\[
q^b \ast (1-q)^a > q^a \ast (1-q)^b
\]

\[
b > a
\]

\[
p \ast q^a \ast (1-q)^b + (1-p) \ast (1-q)^a \ast q^b > p \ast q^a \ast (1-q)^b + (1-p) \ast q^a \ast (1-q)^b
\]

\[
1/() < 1/()
\]

< \ p