

CSC 323 Algorithm Design and Analysis, Spring 2015

Project 7

Due: April 23, 2015: 1 PM

Prove the following three statements. You should submit a written proof (in class) as well as submit a recorded video of your explanation. Your proof needs to be formal (using any of the theorem proving techniques) and you can enhance your explanation with one or more examples.

(1-30 pts) In a directed acyclic graph (DAG), if there is an edge or path from vertex  $u$  to vertex  $v$ , then vertex  $u$  has to always appear somewhere before vertex  $v$  in a topological sort of the DAG.

(2-30 pts) In the examples of the Dijkstra algorithm, we observed that the shortest path weights (minimum path weights) of the vertices optimized in the iterations of the algorithm is in the non-decreasing order. Provide a formal proof of this observation.

(3-40 pts) In a connected weighted graph, a negative weight cycle is said to exist involving two or more vertices  $v_1, v_2, \dots, v_k$  such that the sum of the edges on the cycle  $v_1 - v_2 - v_3 \dots - v_k - v_1$  is negative. Show how the Bellman-Ford algorithm can be used to detect the presence of a negative weight cycle in a connected graph.