

CSC434-Fall2014-information-cascade

$$P[\text{actual} = \text{"blue"}] = 0.2$$
$$P[\text{actual} = \text{"yellow"}] = 0.8$$

$$P[\text{report} = \text{"blue"} \mid \text{actual} = \text{"blue"}] = 0.8$$
$$P[\text{report} = \text{"yellow"} \mid \text{actual} = \text{"blue"}] = 0.2$$

$$P[\text{report} = \text{"yellow"} \mid \text{actual} = \text{"yellow"}] = 0.8$$
$$P[\text{report} = \text{"blue"} \mid \text{actual} = \text{"yellow"}] = 0.2$$

$$P[\text{actual} = \text{"yellow"} \mid \text{report} = \text{"yellow"}] = P[\text{actual} = \text{"yellow"}] * P[\text{report} = \text{"yellow"} \mid \text{actual} = \text{"yellow"}]$$

$$P[\text{report} = \text{"yellow"}]$$

$$P[\text{report} = \text{"yellow"}] = P[\text{actual} = \text{"yellow"}] * P[\text{report} = \text{"yellow"} \mid \text{actual} = \text{"yellow"}] + P[\text{actual} = \text{"blue"}] * P[\text{report} = \text{"yellow"} \mid \text{actual} = \text{"blue"}]$$

$$= 0.2 * 0.8 + 0.8 * 0.2 = 0.32$$

$$P[\text{actual} = \text{"yellow"} \mid \text{report} = \text{"yellow"}] = 0.2 * 0.8 / 0.32 = 0.5$$

$$P[\text{emails are spam}] = 0.4$$
$$P[\text{emails are not spam}] = 0.6$$

$$P[\text{word} = \text{"checkout"} \mid \text{email is spam}] = 0.01$$
$$P[\text{word} = \text{"checkout"} \mid \text{email is NOT spam}] = 0.004$$

$$P[\text{email is spam} \mid \text{word} = \text{"checkout"}] = P[\text{email is spam}] * P[\text{word} = \text{"checkout"} \mid \text{email is spam}] / P[\text{word} = \text{"checkout"}]$$

$$P[\text{word} = \text{"checkout"}] = P[\text{email is spam}] * P[\text{word} = \text{"checkout"} \mid \text{email is spam}] + P[\text{email is not spam}] * P[\text{word} = \text{"checkout"} \mid \text{email is not spam}]$$

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$$\begin{aligned} P[\text{email is not spam}] * P[\text{word} = \\ \text{"checkout"} \mid \text{email is NOT spam}] \\ = 0.4 * 0.01 + 0.6 * 0.004 = 0.0064 \end{aligned}$$

$$P[\text{email is spam} \mid \text{word} = \text{"checkout"}] = 0.4 * 0.01 / 0.0064 = 0.625$$

$$\begin{aligned} P[\text{majority-blue}] &= 1/2 \\ P[\text{majority-red}] &= 1/2 \end{aligned}$$

$$\begin{aligned} P[\text{ball picked is "blue"} \mid \text{urn is "majority-blue"}] &= 2/3 \\ P[\text{ball picked is "red"} \mid \text{urn is "majority-blue"}] &= 1/3 \end{aligned}$$

$$\begin{aligned} P[\text{ball picked is "red"} \mid \text{urn is "majority-red"}] &= 2/3 \\ P[\text{ball picked is "blue"} \mid \text{urn is "majority-red"}] &= 1/3 \end{aligned}$$

$$\begin{aligned} P[\text{urn is majority-blue} \mid \text{ball picked is blue}] \\ = P[\text{urn is majority-blue}] * P[\text{ball picked is blue} \mid \text{urn is} \\ \text{majority blue}] / P[\text{ball picked is blue}] \end{aligned}$$

$$\begin{aligned} P[\text{ball picked is blue}] &= P[\text{urn is majority-blue}] * P[\text{ball} \\ \text{picked is "blue"} \mid \text{urn is "majority-blue"}] + \\ &P[\text{urn is majority-red}] * P[\text{ball} \\ \text{picked is "blue"} \mid \text{urn is "majority-red"}] \\ &= 1/2 * 2/3 + 1/2 * 1/3 = 1/2 \end{aligned}$$

$$P[\text{urn is majority-blue} \mid \text{ball picked is blue}] = 1/2 * 2/3 / 0.5 = 2/3$$

$$\begin{aligned} P[\text{urn is majority-blue} \mid \text{balls picked are blue,blue, red}] \\ = P[\text{urn is majority-blue}] * P[\text{balls picked are blue,} \\ \text{blue,red} \mid \text{urn is majority-blue}] / P[\text{balls picked are} \\ \text{blue,blue,red}] \end{aligned}$$

$$\begin{aligned} P[\text{balls picked are blue, blue,red} \mid \text{urn is majority-blue}] \\ = P[\text{ball picked is blue} \mid \text{urn is} \\ \text{majority-blue}] * \end{aligned}$$

$$P[\text{ball picked is blue} \mid \text{urn is}$$

majority-blue] * P[ball picked is red | urn is

$$\text{majority-blue]} = \frac{2}{3} * \frac{2}{3} * \frac{1}{3} = \frac{4}{27}$$

$$P[\text{balls picked are blue, blue, red} \mid \text{urn is majority-red}] = \frac{1}{3} * \frac{1}{3} * \frac{2}{3} = \frac{2}{27}$$

$$P[\text{balls picked are blue, blue, red}] = \frac{1}{2} * \frac{4}{27} + \frac{1}{2} * \frac{2}{27} = \frac{1}{2} * \frac{6}{27} = \frac{1}{9}$$

$$\frac{2}{27} / \frac{1}{9} = \frac{2}{3}$$

P[see "blue" | hypothesis is "majority-blue"] - high signal (q) > 1/2
 P[see "red" | hypothesis is "majority-blue"] - low signal (1-q)

P[see "red" | hypothesis is "majority-red"] - low signal (q) > 1/2
 P[see "blue" | hypothesis is "majority-red"] - high signal (1-q)

S = sequence of 'a' high signals and sequence of 'b' low signals

$$P[\text{majority-blue} \mid S] = \frac{P[\text{majority-blue}] * P[S \mid \text{majority-blue}]}{P[\text{majority-blue}] * P[S \mid \text{majority-blue}] + P[\text{majority-red}] * P[S \mid \text{majority-red}]}$$

$$= p * q^a * (1-q)^b + (1-p) * (1-q)^a * q^b$$

$$P[\text{majority-blue} \mid S] = \frac{p * q^a * (1-q)^b}{p * q^a * (1-q)^b + (1-p) * (1-q)^a * q^b}$$

$$q^a b$$

a > b

Since q > 1/2

$$q^a * (1-q)^b > (1-q)^a * q^b$$

$$p * q^a * (1-q)^b + (1-p) * (1-q)^a * q^b < p * q^a * (1-q)^b + (1-p) * q^a * (1-q)^b$$

$$1/() > 1/()$$

$$P[\text{majority-blue} \mid S] > p$$

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$$a < b$$

$$P[\text{majority-blue} \mid S] = \frac{p * q^a * (1-q)^b}{p * q^a * (1-q)^b + (1-p) * (1-q)^a * q^b}$$

$$q^b * (1-q)^a > q^a * (1-q)^b$$

$$b > a$$

$$b > a$$

$$p * q^a * (1-q)^b + (1-p) * (1-q)^a * q^b > p * q^a * (1-q)^b + (1-p) * q^a * (1-q)^b$$

$$1/() < 1/()$$

$$< p$$