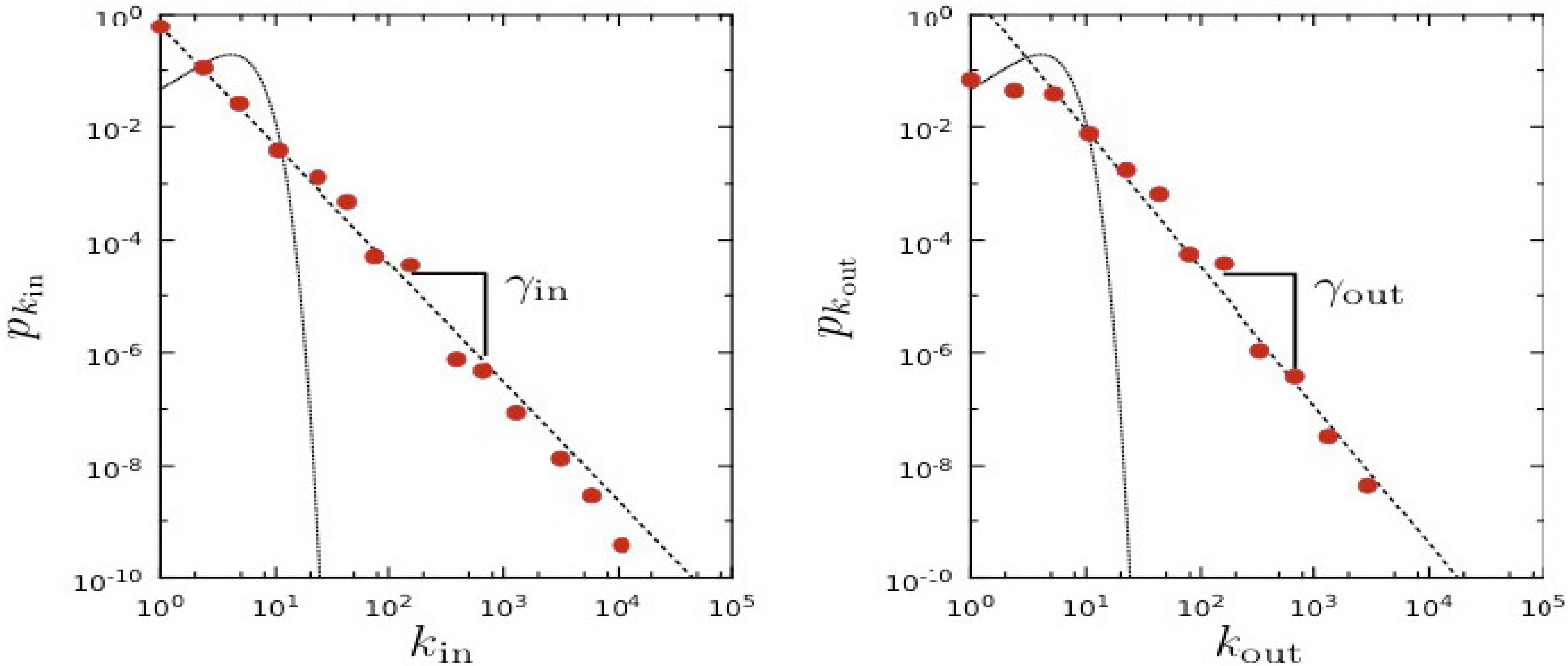
Scale-Free Networks

Dr. Natarajan Meghanathan
Professor
Department of Computer Science
Jackson State University, Jackson, MS
E-mail: natarajan.meghanathan@jsums.edu

Degree Distribution of the WWW

- The degree distribution of the WWW is best modeled according to the Power Law
  - $p_k \sim k^{-\gamma}$
  - $\log(p_k) \sim (-\gamma)*(\log k)$
  - The slope values for γ are: $\gamma_{in} = 2.1$ and $\gamma_{out} = 2.45$.
- A random network model fit (Poisson distribution) for the WWW based on a $<k_{in}> = <k_{out}> = 4.6$ is also shown below.

Both the in- and out- degree distributions are best approximated with a power law.



Power Law Distribution

γ -->

k	1.9	2	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3	3.1
0													
1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	0.27	0.25	0.23	0.22	0.2	0.19	0.1768	0.1649	0.1539	0.1436	0.134	0.125	0.117
3	0.12	0.11	0.1	0.09	0.08	0.07	0.0642	0.0575	0.0515	0.0461	0.0413	0.037	0.033
4	0.07	0.06	0.05	0.05	0.04	0.04	0.0313	0.0272	0.0237	0.0206	0.0179	0.0156	0.014
5	0.05	0.04	0.03	0.03	0.02	0.02	0.0179	0.0152	0.013	0.011	0.0094	0.008	0.007
6	0.03	0.03	0.02	0.02	0.02	0.01	0.0113	0.0095	0.0079	0.0066	0.0055	0.0046	0.004
7	0.02	0.02	0.02	0.01	0.01	0.01	0.0077	0.0063	0.0052	0.0043	0.0035	0.0029	0.002
8	0.02	0.02	0.01	0.01	0.01	0.01	0.0055	0.0045	0.0036	0.003	0.0024	0.002	0.002
9	0.02	0.01	0.01	0.01	0.01	0.01	0.0041	0.0033	0.0027	0.0021	0.0017	0.0014	0.001
10	0.01	0.01	0.01	0.01	0.01	0	0.0032	0.0025	0.002	0.0016	0.0013	0.001	8E-04

kp(k)

1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	0.54	0.5	0.47	0.44	0.41	0.38	0.3536	0.3299	0.3078	0.2872	0.2679	0.25	0.233
3	0.37	0.33	0.3	0.27	0.24	0.21	0.1925	0.1724	0.1545	0.1384	0.124	0.1111	0.1
4	0.29	0.25	0.22	0.19	0.16	0.14	0.125	0.1088	0.0947	0.0825	0.0718	0.0625	0.054
5	0.23	0.2	0.17	0.14	0.12	0.11	0.0894	0.0761	0.0648	0.0552	0.047	0.04	0.034
6	0.2	0.17	0.14	0.12	0.1	0.08	0.068	0.0569	0.0475	0.0397	0.0332	0.0278	0.023
7	0.17	0.14	0.12	0.1	0.08	0.07	0.054	0.0444	0.0366	0.0301	0.0248	0.0204	0.017
8	0.15	0.13	0.1	0.08	0.07	0.05	0.0442	0.0359	0.0292	0.0237	0.0192	0.0156	0.013
9	0.14	0.11	0.09	0.07	0.06	0.05	0.037	0.0297	0.0239	0.0192	0.0154	0.0123	0.01
10	0.13	0.1	0.08	0.06	0.05	0.04	0.0316	0.0251	0.02	0.0158	0.0126	0.01	0.008
Avg	3.22	2.93	2.68	2.47	2.29	2.13	1.9953	1.8793	1.7789	1.6918	1.616	1.5498	1.492

Power Law vs. Poisson Distribution

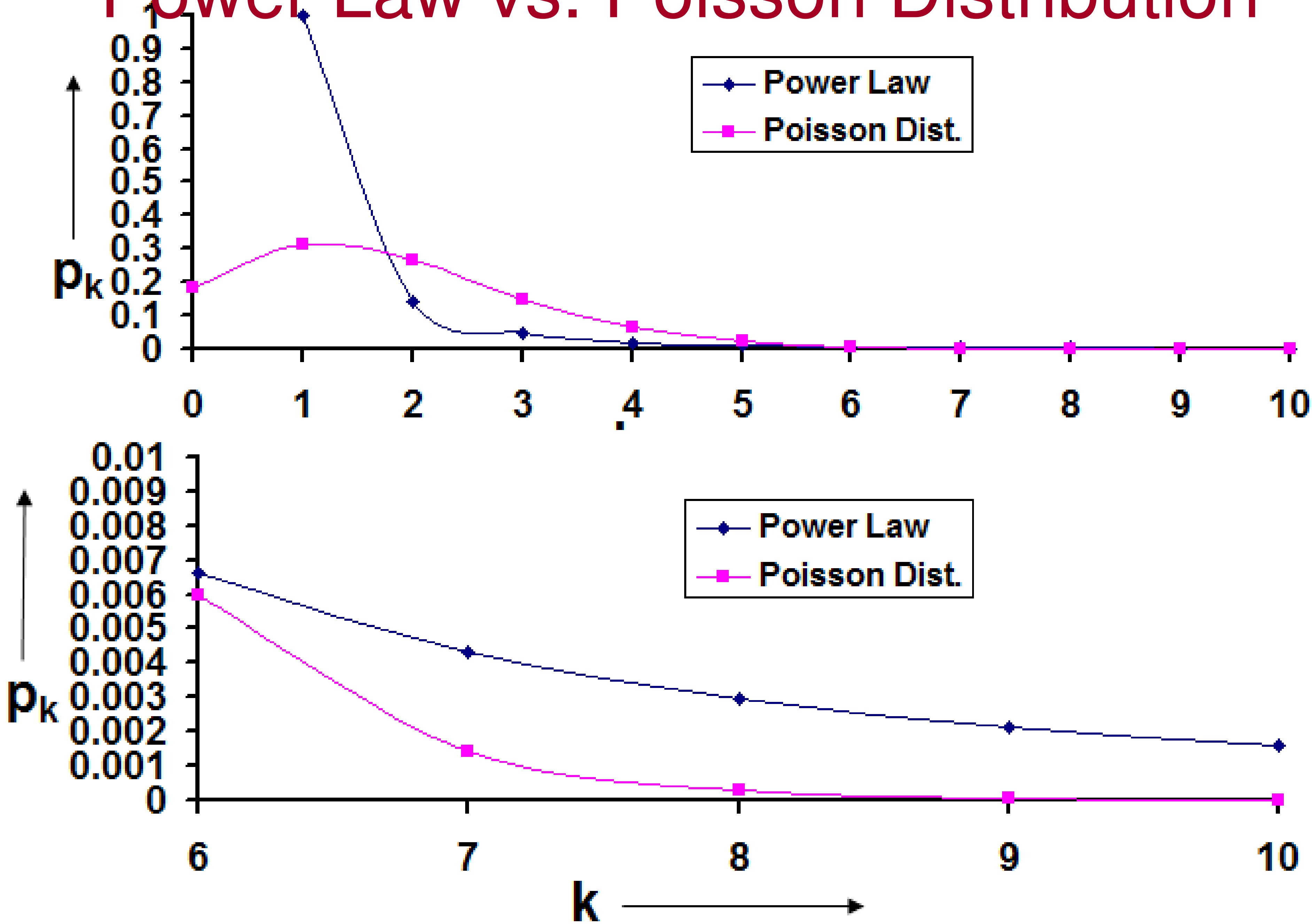
k	$p_k = k^{-2.8}$	$k*p_k$
0		
1	1	1
2	0.143587294	0.287175
3	0.046138183	0.138415
4	0.020617311	0.082469
5	0.011037837	0.055189
6	0.006624857	0.039749
7	0.004302546	0.030118
8	0.002960384	0.023683
9	0.002128732	0.019159
10	0.001584893	0.015849
<k>		**1.691805**

k	$p_k = (e^{-\langle k \rangle} * \langle k \rangle^k)/k!$	$k*p_k$
0	0.185473513	0
1	0.313450237	0.31345
2	0.264865451	0.529731
3	0.149207537	0.447623
4	0.063040184	0.252161
5	0.021307582	0.106538
6	0.006001636	0.03601
7	0.001448966	0.010143
8	0.000306094	0.002449
9	5.74777E-05	0.000517
10	9.71373E-06	9.71E-05
<k>		**1.698718**

Power Law: About 0.15% of the nodes in the network have a degree of 10. (power-law distribution have a long tail).

Poisson: Only about 0.00097% of the nodes in the network have a degree of 10.

Power Law vs. Poisson Distribution



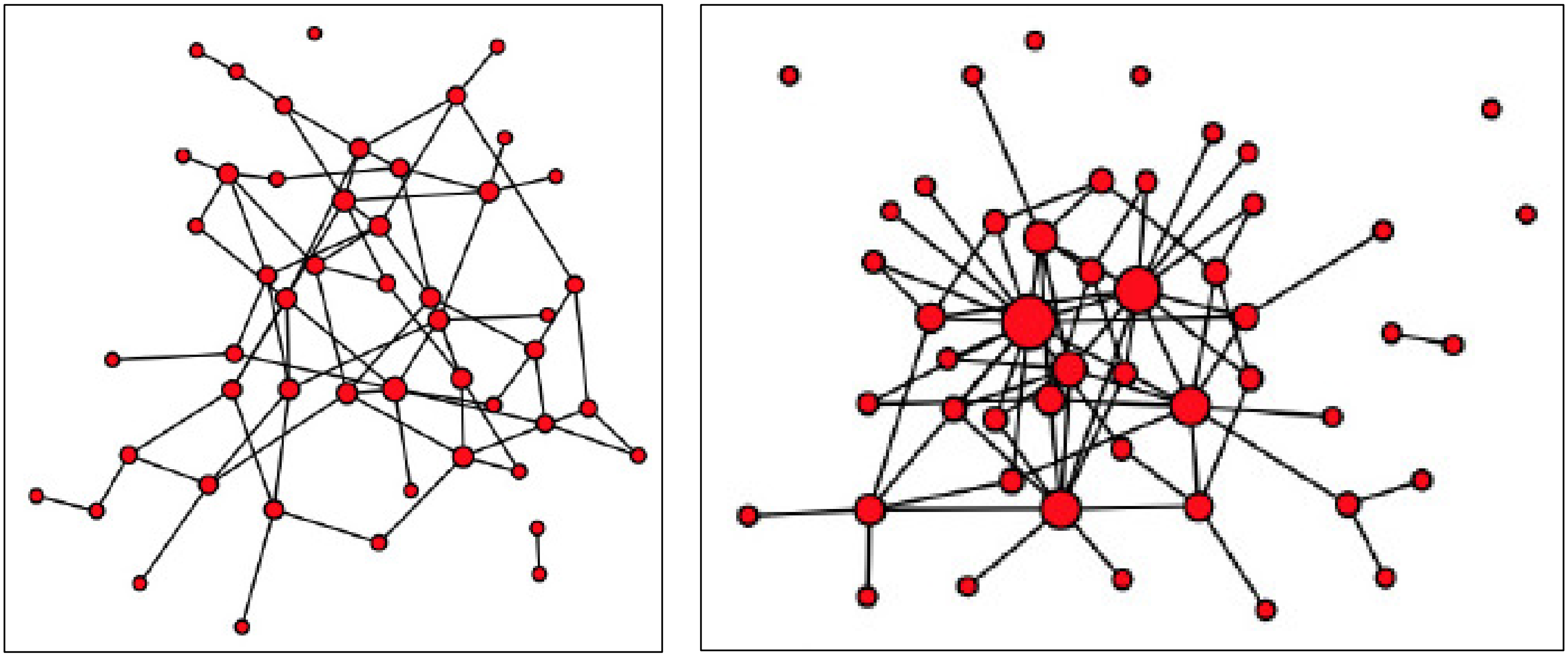
Power Law vs. Poisson Distribution

- For small k, the power law is above the Poisson function, hence a scale-free network has a large number of small degree nodes that are virtually absent in a random network.

- For the *k* in the vicinity of <k>, the Poisson distribution is above the Power Law, indicating that in a random network, most nodes have degree k ≈ <k>.

- For large k, the power law curve is again above the Poisson curve, indicating the Probability of observing a high-degree node is several orders of magnitudes higher in a scale-free network than in a random network.

Random Network vs. Scale-Free Network



Source: Figure 4.4 Barabasi

A random network with <k> = 3 and N = 50 nodes, illustrates that most nodes have comparable degree k = <k>.
A scale-free network with <k> = 3, illustrating that numerous small-degree nodes coexist with a few highly connected hubs.

Power Law in Reality

- Economy:
  - In Italy, a 19th century economist (Pareto) noticed that a few wealthy individuals earned most of the money, while the remaining population earned small amounts.
    - About 80% of the money is earned by only 20% of the population.
  - In US, during the 2009 economic crisis, the Occupy Wall Street movement highlighted the fact that 1% of the population earned a disproportionate 15% of the income.

- Management: 80% of the decisions taken in a meeting are made only during 20% of the working time.

- Networks:
  - 80% of the web point to only 15% of the web pages
  - 80% of the citations belong to only 38% of the scientists

Scale-Free Networks

- Networks whose degree distribution follows a Power Law are said to be scale-free networks.
- The power law degree distribution can be defined in both discrete and continuous formalisms.
  - The scale-free property is independent of the formalism used to describe the degree distribution.

Discrete Formalism

$$p_k = Ck^{-\gamma}$$

$$\sum_{k=1}^{\infty} p_k = 1$$

$$C\sum_{k=1}^{\infty} k^{-\gamma} = 1$$

$$C = \frac{1}{\sum_{k=1}^{\infty} k^{-\gamma}} = \frac{1}{\zeta(\gamma)}$$

$$p_k = \frac{k^{-\gamma}}{\zeta(\gamma)}$$

$$\zeta(\gamma) = \sum_{k=1}^{\infty} k^{-\gamma}$$

$\zeta(\gamma)$ is called the Riemann-Zeta Function

Note that p_k (as defined above) diverges for k = 0 (isolated nodes). For such nodes, we need to specify p_0.

Continuum Formalism

$$p(k) = Ck^{-\gamma}$$

$$\int_{k_{min}}^{\infty} p(k)dk = 1$$

$$C = \frac{1}{\int_{K_{min}}^{\infty} k^{-\gamma} dk} = (\gamma - 1)K_{min}^{\gamma - 1}$$

$$\boxed{p(k) = (\gamma - 1)k_{min}^{\gamma - 1}\, k^{-\gamma}}$$

K_{min} is the smallest degree for which the power law for the discrete formalism holds.
Note that p_k encountered in the discrete formalism provides the probability that a randomly chosen node has degree k. In contrast, only the integral of *p(k)* encountered in the continuum formalism has a physical interpretation.

$$\int_{k_1}^{k_2} p(k)dk$$

probability that a randomly chosen node has degree between k_1 and k_2.

Hubs: Exponential Distribution

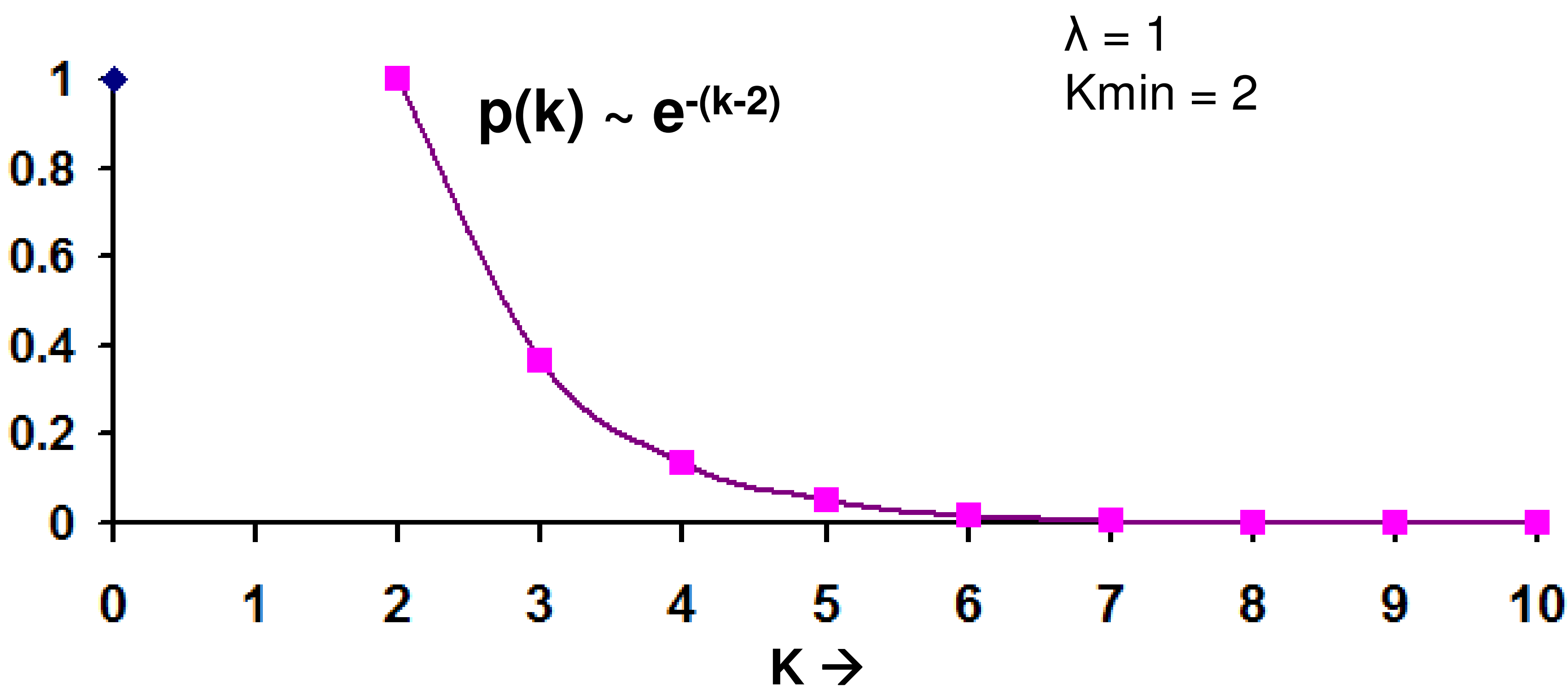
- Assume the distribution of node degree follows an exponential distribution pk = Ce $^{-\lambda k}$. Let kmin indicate the minimum node degree. Then, as per the normalization condition: $$\int_{k_{min}}^{\infty} p(k)dk = 1$$
- yielding, C = λe$^{\lambda kmin}$.
- To calculate kmax (the maximum degree of a node), an upper bound for the degree of a hub node, we assume that in a network of N nodes, there is at most one node in the (kmax, ∞) regime. In other words, the probability to observe a node whose degree is kmax or above is 1/N.

$$\int_{k_{max}}^{\infty} p(k)dk = \frac{1}{N}$$

Solving this integral for kmax,
We obtain:

$$k_{max} = k_{min} + \frac{\ln N}{\lambda}$$

Exponential Distribution



Hubs: Exponential Distribution

$$p_k = Ce^{-\lambda k}$$

$$\int_{k_{\min}}^{\infty} p_k dk = 1$$

$$\int_{k_{\min}}^{\infty} Ce^{-\lambda k} dk = 1$$

$$C\left\{\frac{e^{-\lambda k}}{-\lambda}\right\}_{k_{\min}}^{\infty} = 1$$

$$\frac{C}{\lambda}\left\{e^{-\lambda k_{\min}}\right\} = 1$$

$$C = \lambda e^{\lambda k_{\min}}$$

$$p_k = \lambda e^{\lambda k_{\min}} e^{-\lambda k} = \lambda e^{-\lambda(k - k_{\min})}$$

Hubs: Exponential Distribution

$$\int_{k_{max}}^{\infty} p_k dk = \frac{1}{N} \qquad \int_{k_{max}}^{\infty} Ce^{-\lambda k} dk = \frac{1}{N}$$

$$C\left\{\frac{e^{-\lambda k}}{-\lambda}\right\}_{k_{max}}^{\infty} = \frac{1}{N} \qquad \frac{C}{\lambda}\left\{e^{-\lambda k_{max}}\right\} = \frac{1}{N}$$

$$e^{\lambda k_{max}} = \frac{NC}{\lambda} \qquad \lambda k_{max} = \ln\frac{NC}{\lambda}$$

$$k_{max} = \frac{1}{\lambda}\left\{\ln\frac{NC}{\lambda}\right\} = \frac{1}{\lambda}\left\{\ln\left(Ne^{\lambda k_{min}}\right)\right\}$$

$$k_{max} = \frac{1}{\lambda}\left\{\ln N + \ln\left(e^{\lambda k_{min}}\right)\right\} \qquad k_{max} = k_{min} + \frac{\ln N}{\lambda}$$

Hubs: Scale-Free Property

$$p_k = Ck^{-\gamma}$$

$$\int_{k_{\min}}^{\infty} p_k dk = 1$$

$$\int_{k_{\min}}^{\infty} Ck^{-\gamma} dk = 1$$

$$C\left\{\frac{k^{-\gamma+1}}{-\gamma+1}\right\}_{k_{\min}}^{\infty} = 1$$

$$\frac{C}{\gamma-1}\left\{\frac{1}{k_{\min}^{\gamma-1}}\right\} = 1$$

$$C = (\gamma-1)k_{\min}^{\gamma-1}$$

$$p_k = (\gamma-1)k_{\min}^{\gamma-1}k^{-\gamma}$$

Hubs: Scale-Free Property

$$\int_{k_{\max}}^{\infty} p_k \, dk = \frac{1}{N}$$

$$\int_{k_{\max}}^{\infty} C k^{-\gamma} \, dk = \frac{1}{N}$$

$$C \left\{ \frac{k^{-\gamma+1}}{-\gamma+1} \right\}_{k_{\max}}^{\infty} = \frac{1}{N}$$

$$\frac{C}{\gamma-1} \left\{ \frac{1}{k_{\max}^{\gamma-1}} \right\} = \frac{1}{N}$$

$$k_{\max}^{\gamma-1} = \frac{NC}{\gamma-1} = N k_{\min}^{\gamma-1}$$

$$k_{\max} = k_{\min} N^{\frac{1}{\gamma-1}}$$

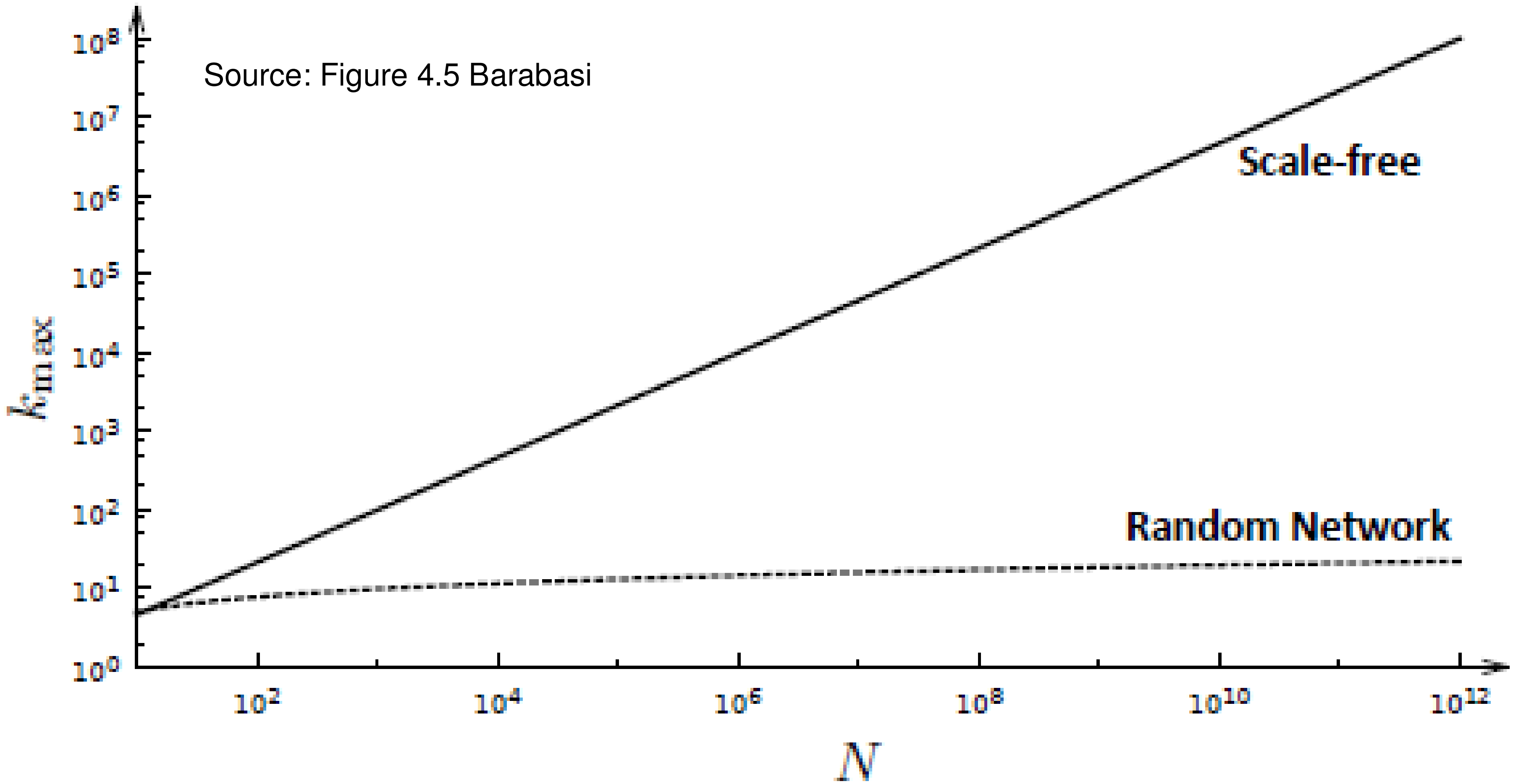
Hubs: Exponential vs. Scale-Free Networks

$$k_{max} = k_{min} + \frac{\ln N}{\lambda}$$

$$k_{max} \sim k_{min} \; N^{\frac{1}{\gamma - 1}}$$

- Note that ln*N* is a slow function of the system size, *N*.
- The above equation tells us that the maximum degree will not be very different from kmin.
- For a Poisson distribution (the calculation is a bit more involved), kmax (as a function of N) grows even slower than the logarithmic dependence on N as shown above for the exponential distribution.
- For a scale-free network, the value of kmax is related to kmin and N as shown above, illustrating that the larger a network, the larger is the degree of its biggest hub.
- The polynomial dependence of kmax on N for a scale-free network implies that in a large-scale network, there can be orders of magnitude differences in size between the smallest node (kmin) and the biggest hub (kmax).

Hubs are Large in Scale-free Networks

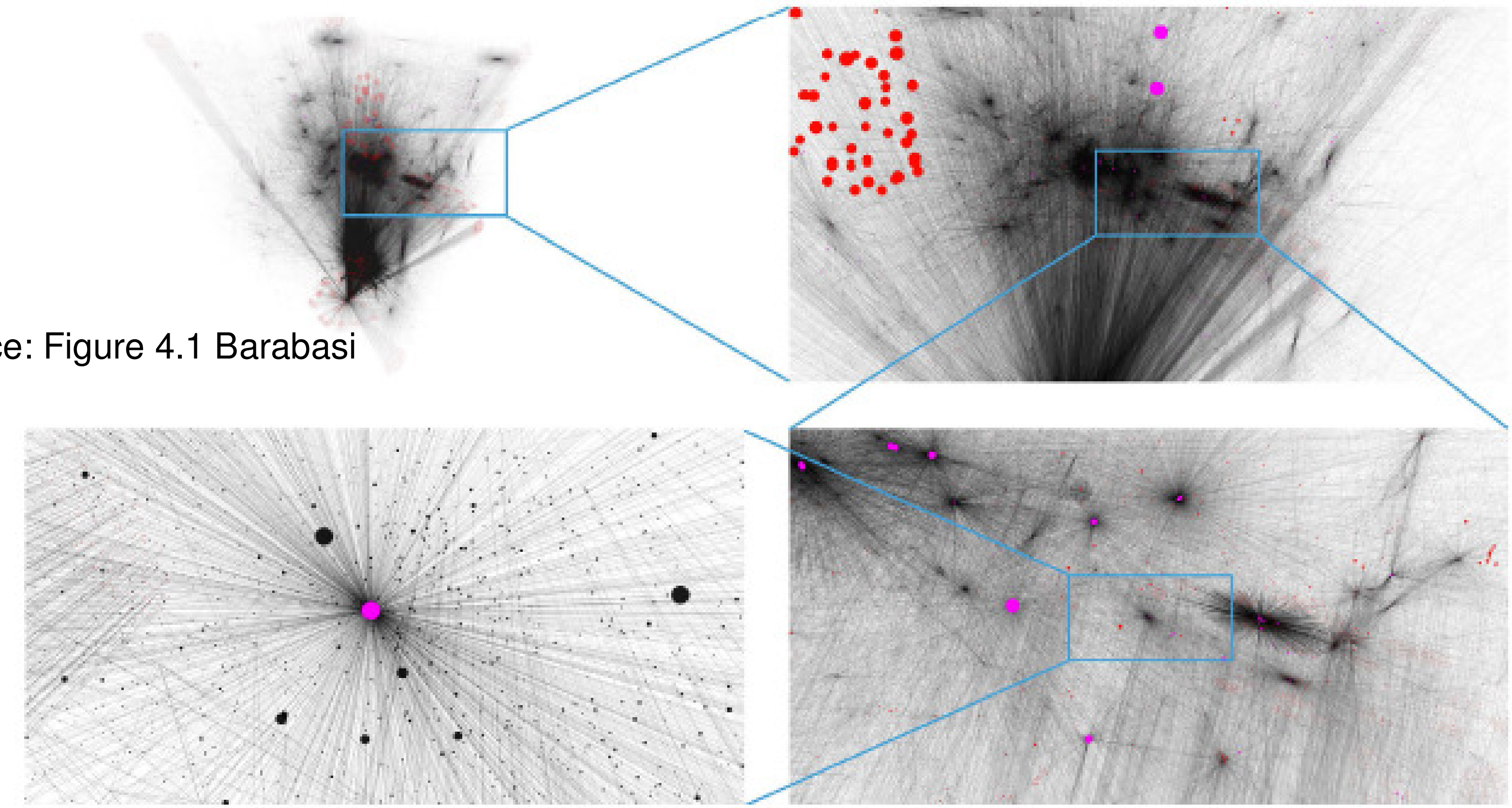


<k> = 3 and γ = 2.5

The above figure illustrates that hubs in a scale-free network are several orders of magnitude larger than hubs in a random network of similar N and <k>.

Hubs in WWW: Exponential vs. Scale-free

- Consider a WWW sample of $N = 3 * 10^5$ nodes. With a kmin = 1, λ = 1, and γ = 2.1,
  - If the degree distribution were to be exponential, kmax ≈ 13.
  - If the degree distribution is scale-free, kmax = 95,000.

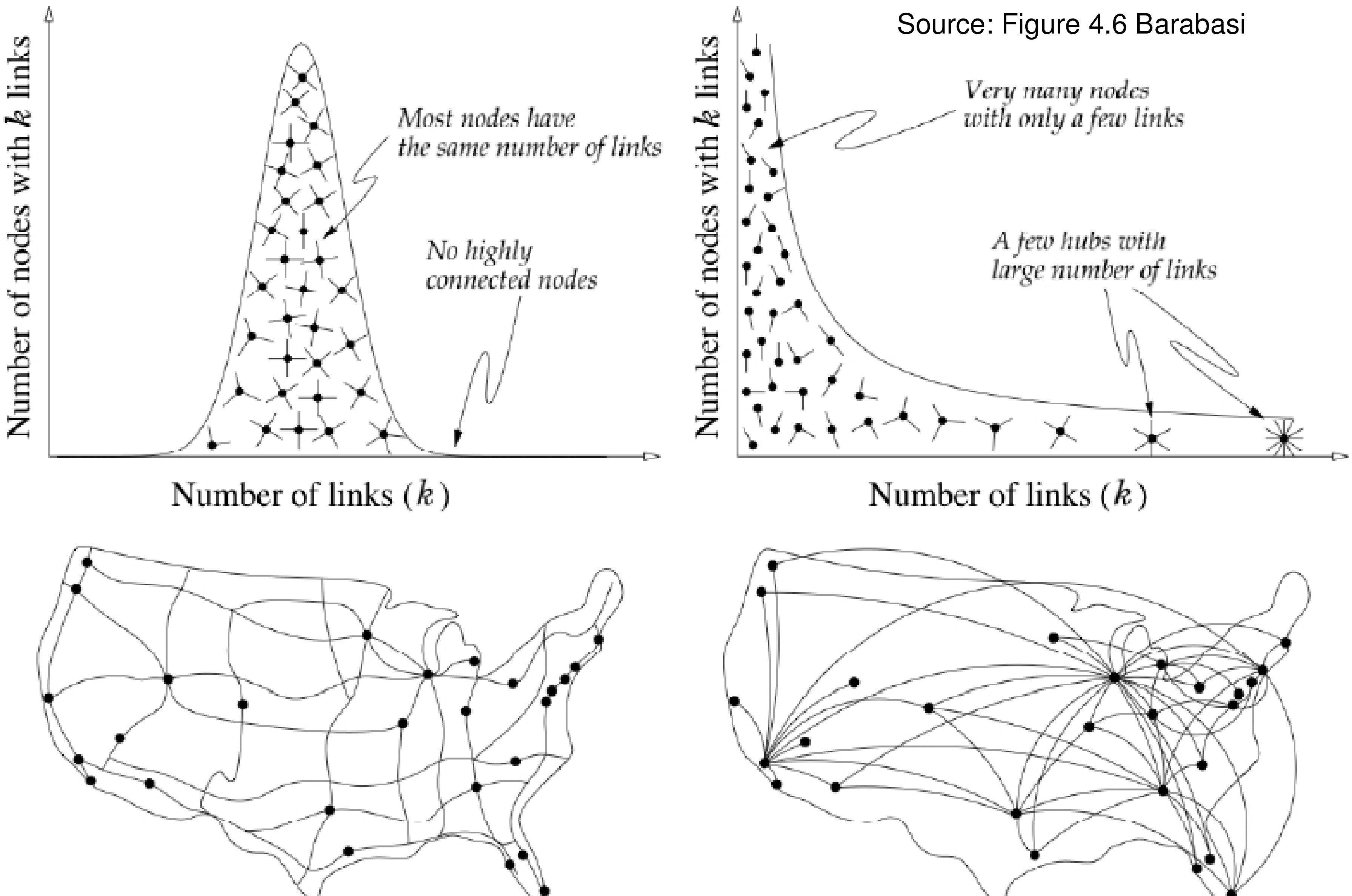


Source: Figure 4.1 Barabasi

In a random network – hubs are forbidden (most nodes have comparable degrees); In a scale-free network, hubs occur naturally (are expected to occur).

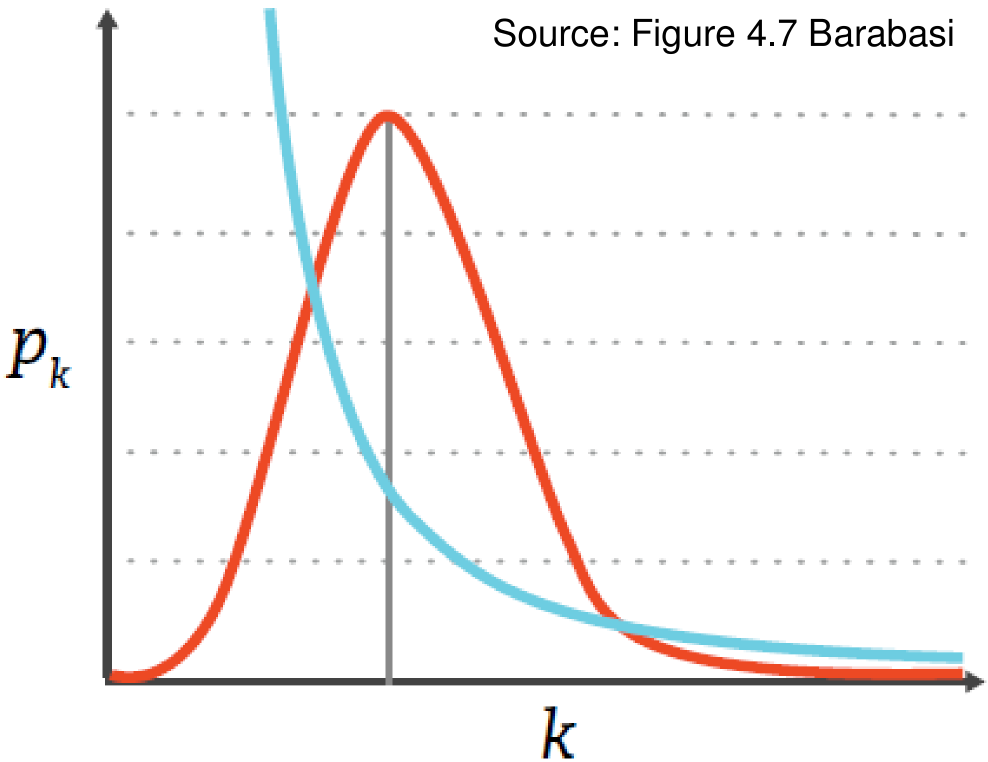
Road Network vs. Air-traffic Network

Source: Figure 4.6 Barabasi



there are no major cities with hundreds of highways; but, certain cities are airport hubs

Scale-free networks lack an intrinsic scale



Source: Figure 4.7 Barabasi

Random network
Randomly chosen node: $k = \langle k \rangle \pm \langle k \rangle^{1/2}$
Scale: $\langle k \rangle$

Scale-free network
Randomly chosen node: $k = \langle k \rangle \pm \infty$
$\langle k \rangle$ is meaningless as 'scale'

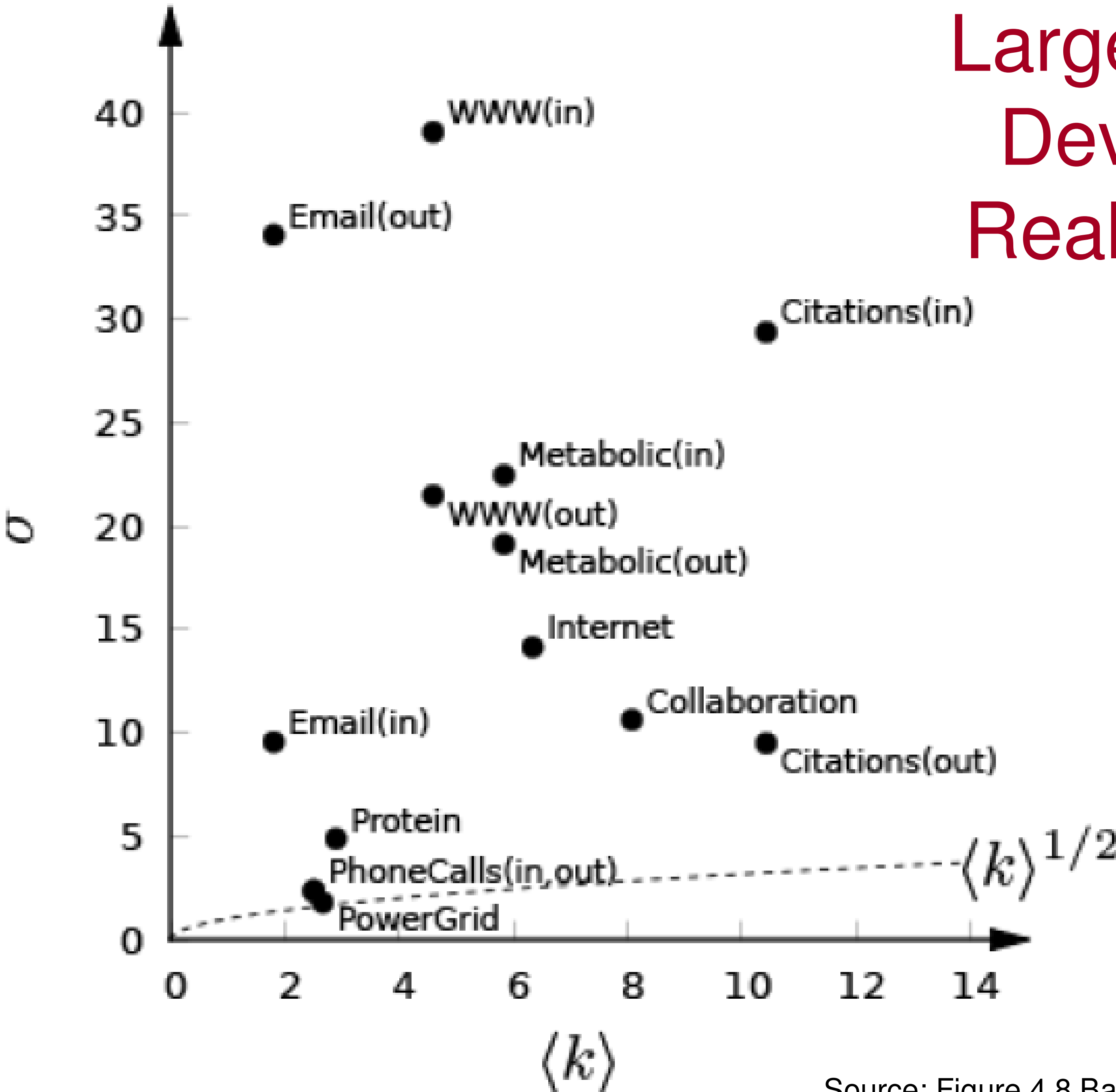
- For any bounded distribution (e.g. a Poisson or
- a Gaussian distribution) the degree of a randomly chosen node will be in the vicinity of <k>. Hence <*k*> serves as the network's scale.
- In a scale-free network the second moment diverges, hence the degree of a randomly chosen node can be arbitrarily different from <*k*>. Thus, a scale-free network lacks an intrinsic scale (and hence gets its name).

Scale-free nature of real networks

NETWORK	NL		$\langle k \rangle$ $\langle k_{in} \rangle = \langle k_{out} \rangle$	σ_{in}	σ_{out}	σ	γ_{in}	γ_{out}	γ
Internet	192,244	609,066	6.34	-	-	14.14	-	-	3.42*
WWW	325,729	1,497,134	4.60	39.05	21.48	-	2.31	2.00	-
Power Grid	4,941	6,594	2.67	-	-	1.79	-	-	Exp.
Mobile Phone Calls	36,595	91,826	2.51	2.39	2.32	-	4.69*	5.01*	-
Email	57,194	103,731	1.81	9.56	34.07	-	3.43*	2.03	-
Science Collaboration	23,133	93,439	8.08	-	-	10.63	-	-	3.35
Actor Network	702,388	29,397,908	83.71	-	-	200.86	-	-	2.12
Citation Network	449,673	4,689,479	10.43	29.37	9.49	-	3.03**	4.00	-
E. Coli Metabolism	1,039	5,802	5.58	22.46	19.12	-	2.43	2.90	-
Yeast Protein Interactions	2,018	2,930	2.90	-	-	4.88	-	-	2.89*

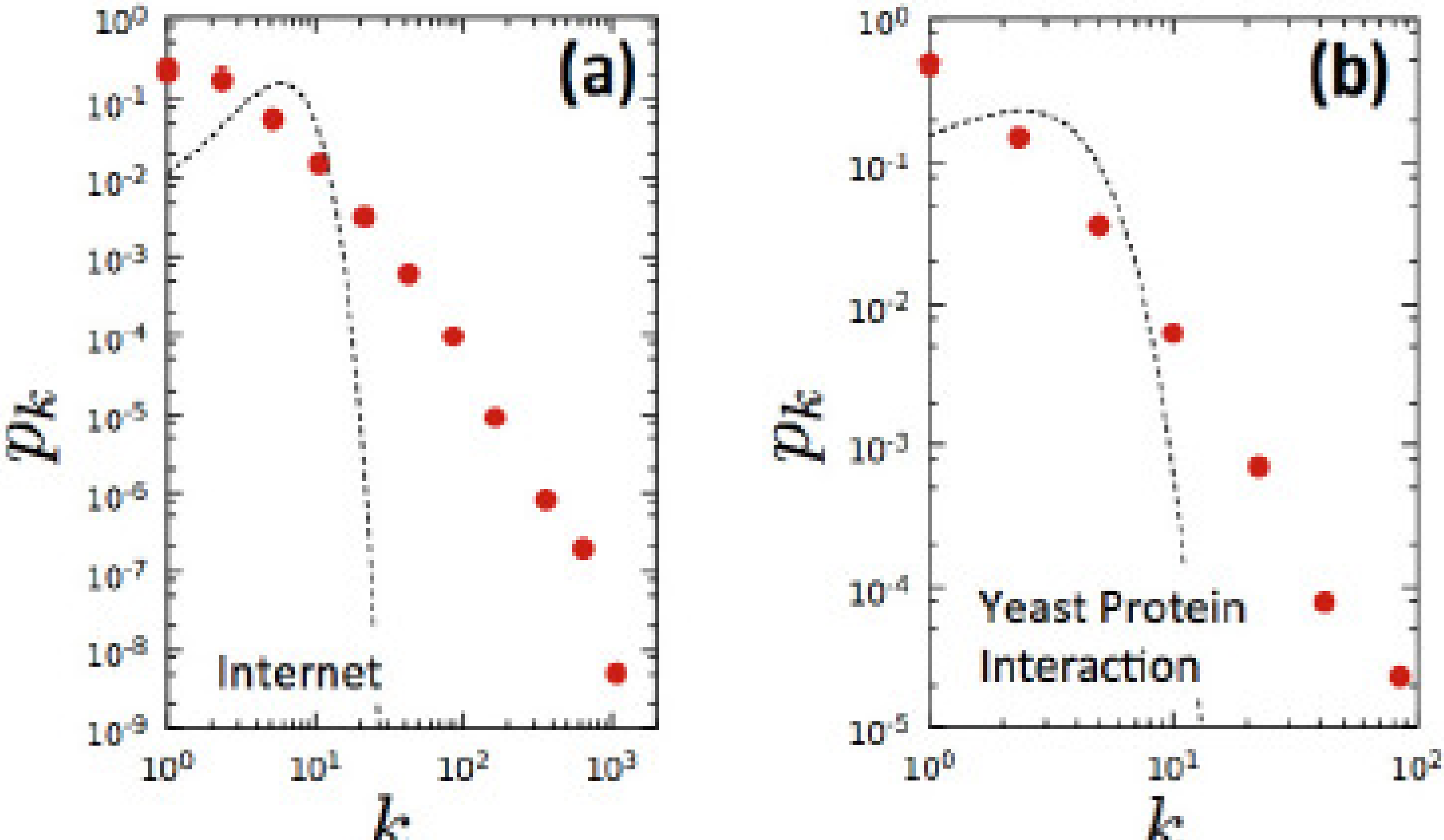
Source: Table 4.1 Barabasi

Large Standard Deviation for Real Networks



Source: Figure 4.8 Barabasi

Many real networks are scale-free



The dotted line shows the Poisson distribution with the same <k> as that of the real network.

Source: Figure 4.9b Barabasi

Requirement for a Scale-free network to emerge

- For the scale-free property to emerge, the nodes need to have the capacity to link to an arbitrary number of other nodes.

- In general, the scale-free property is absent in systems that have a limitation in the number of links a node can have, as such limitations limit the size of the hubs.

Average Distance: Power-Law

$$
d \sim \begin{cases} \text{const.} & \text{if } \gamma = 2, \\ \dfrac{\ln \ln N}{\ln(\gamma - 1)} & \text{if } 2 < \gamma < 3, \\ \dfrac{\ln N}{\ln \ln N} & \text{if } \gamma = 3, \\ \ln N & \text{if } \gamma > 3. \end{cases}
$$

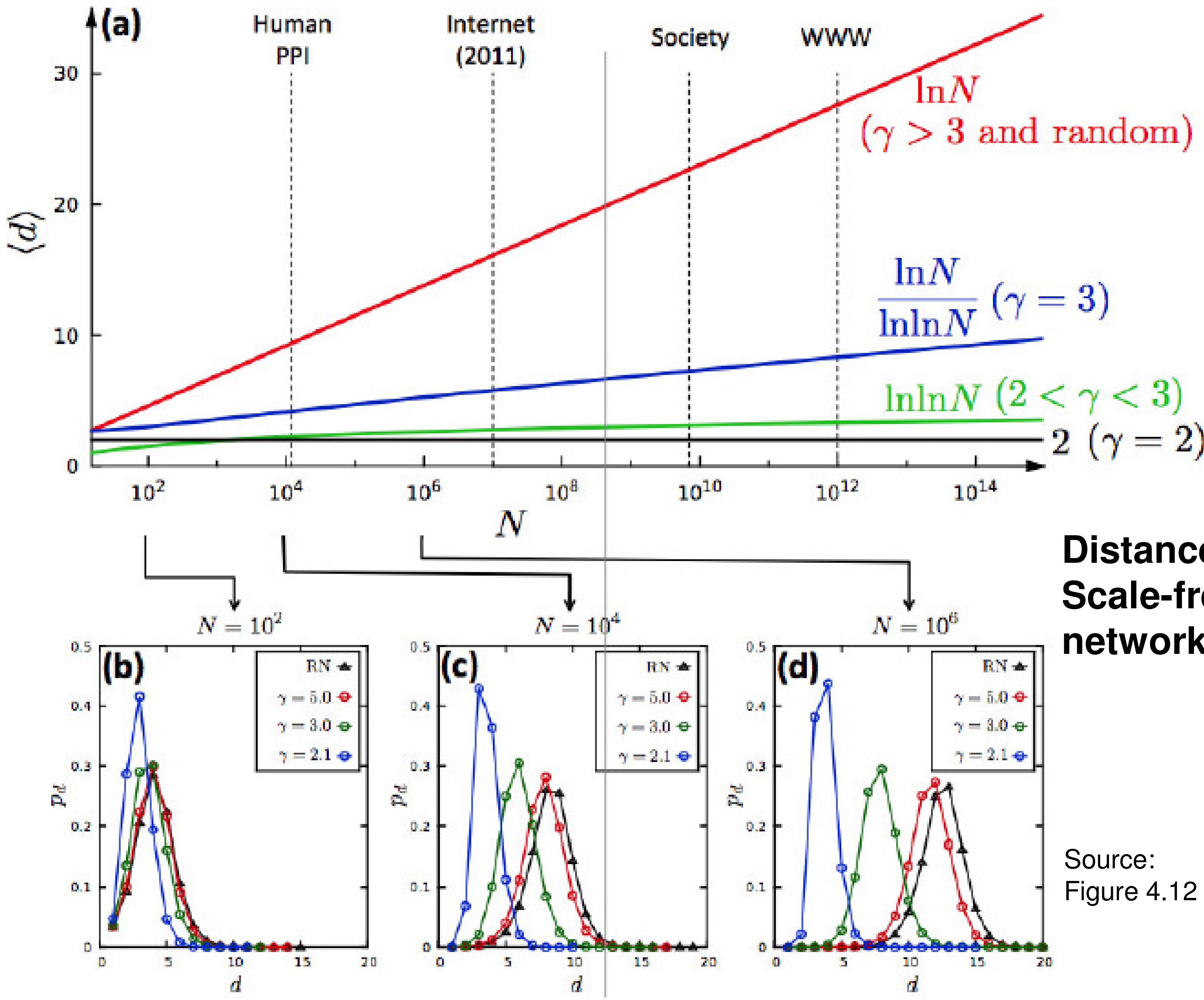
Anomalous regime: Hub and spoke configuration; average distance independent of N.

Ultra small world regime
Hubs still reduce the path length

the lnN dependence on N (as in random networks) starts

Small world property: Hubs are not sufficiently large and numerous to have impact on path length

The scale-free property shrinks the average path lengths as well as changes the dependence of <d> on the system size. The smaller γ, the shorter are the distances between the nodes.



Distances in Scale-free networks

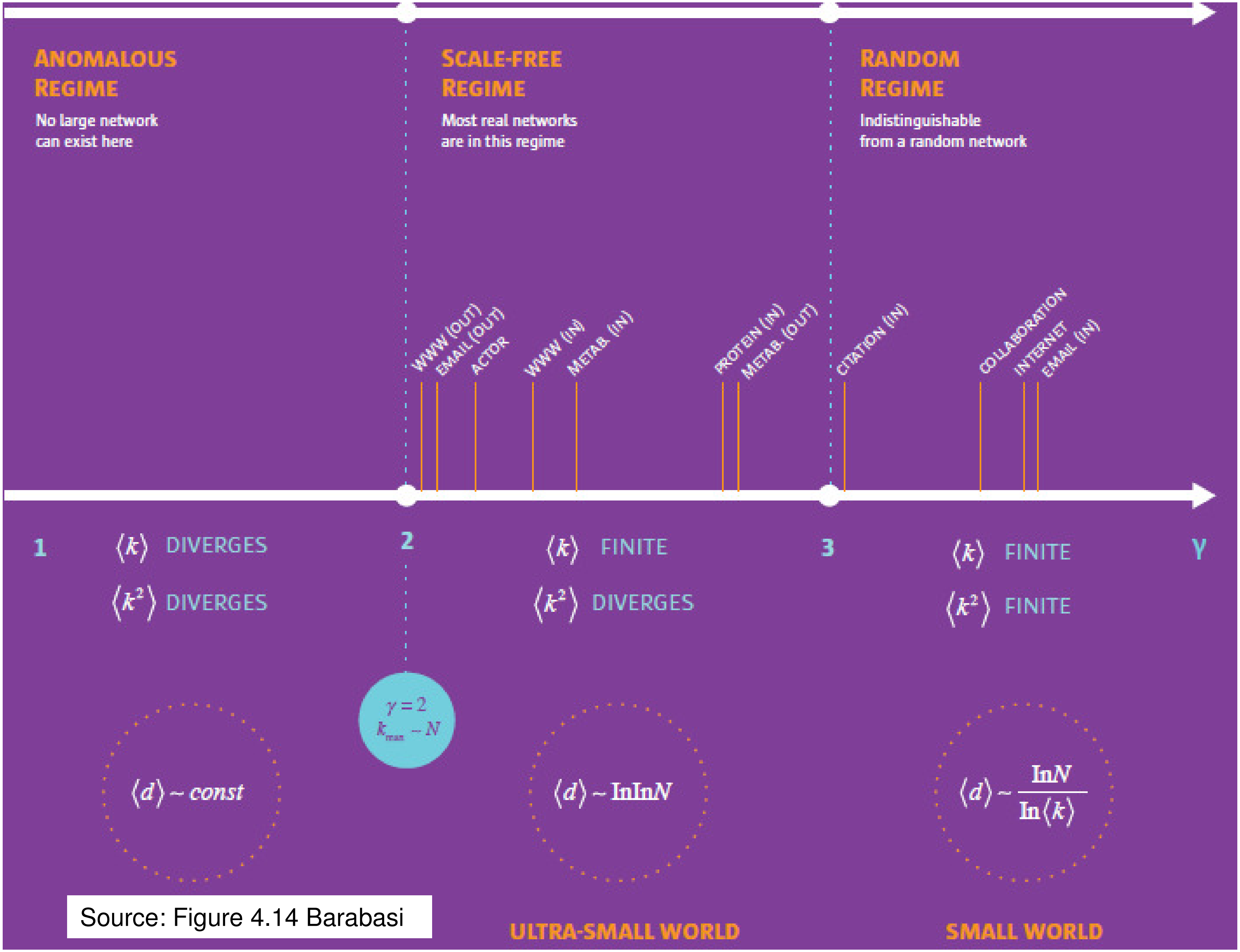
Source:
Figure 4.12 Barabasi

The Role of the Degree Exponent

- Anomalous Regime ($\gamma \leq 2$): For values of $\gamma < 2$, the value of $1/(\gamma-1)$ is greater than 1.
  - This implies that the largest hub should have a degree greater than N.

$$k_{\max} = k_{\min} N^{\frac{1}{\gamma-1}}$$

    - For this, the hub should have self-loops and/or multiple links between the hub and the other nodes.
    - Neither of these are common in real-networks. Hence, it is very rare to find real networks whose degree distribution fit to a power law with $\gamma < 2$.
- Scale-Free Regime ($2 < \gamma < 3$): The first moment $\langle k \rangle$ is finite; whereas the second and higher moments diverge as $N \rightarrow \infty$.
  - Kmax grows with the size of the network with exponent $1/(\gamma-1)$ of value less than 1.
- Random Network Regime ($\gamma > 3$): The first and second moments are finite.
  - For large γ, the degree distribution pk decays sufficiently fast to make the hubs smaller and less numerous (characteristic of random networks): uncharacteristic of real-networks.



Source: Figure 4.14 Barabasi

Real Networks: Scale-Free.. Why?

- Growth: While the random network model assumes that the number of nodes is fixed (time invariant), real networks are the result of a growth process that continuously increases the number of nodes.

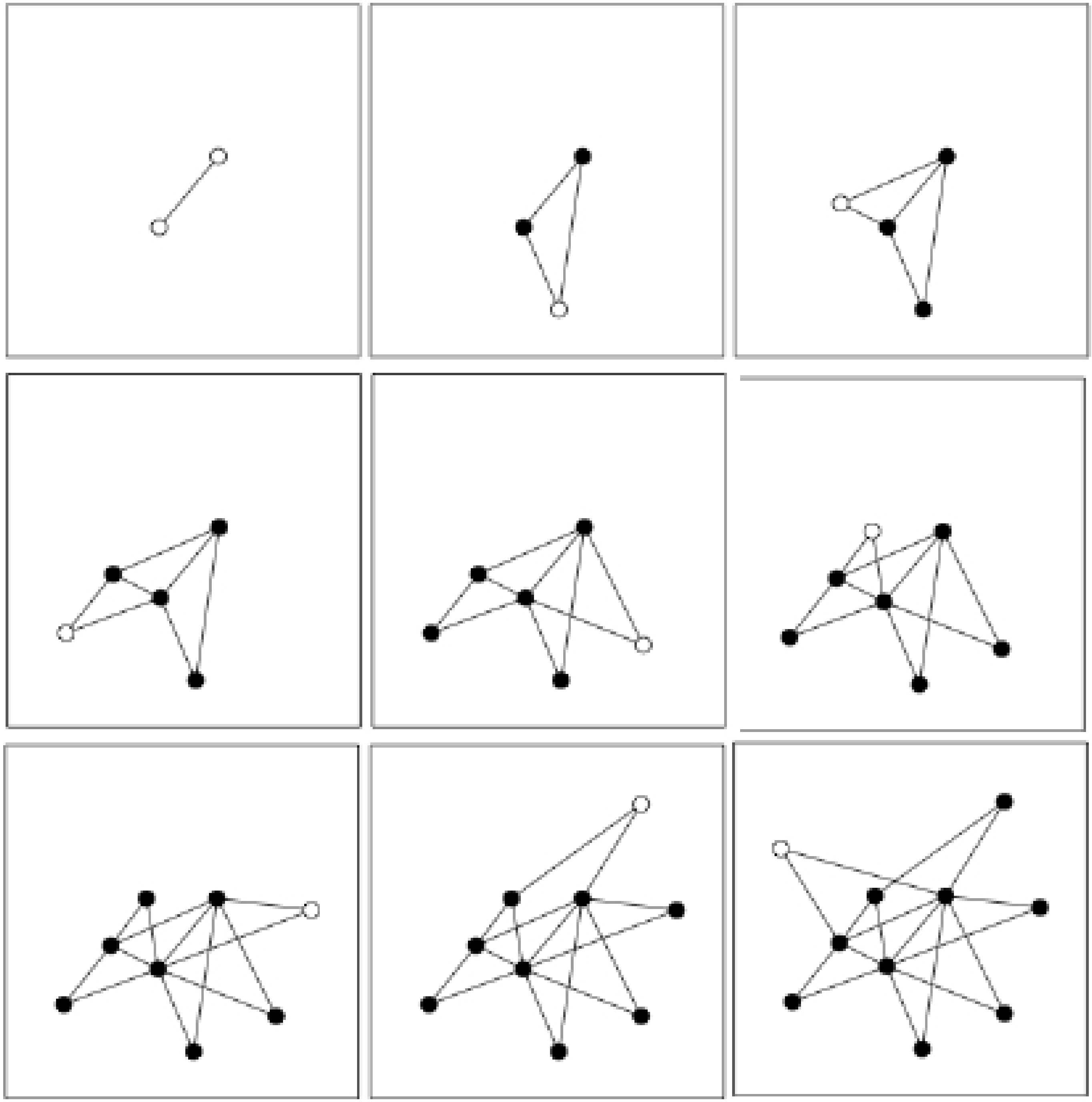
- Preferential Attachment: While nodes in a random network randomly chose their interaction partner, in real networks new nodes prefer to link to the more connected nodes.

Barabasi Albert (BA) Model

- BA model is a model for generating networks with power-law degree distribution.
- The model is defined as follows:
  - We start with m_0 nodes, the links between which are chosen arbitrarily, as long as each node has at least one link.
  - The network develops as per the following growth and preferential attachment properties:
    - Growth: At each time step, we add a new node with m ($\leq m_0$) links that connect the new node to m nodes already in the network.
    - Preferential Attachment: The probability π(k) that one of the links of the new node connects to node *i* depends on the degree k_i of node *i* as:

a node with larger degree has good chances of getting connected to even more nodes.

$$\Pi(k_i) = \frac{k_i}{\sum_j k_j}$$



BA Model Example (m = 2)

Source: Figure 5.4 Barabasi

Time Dependent Degree of a Node

- In the BA model, a node has a chance to increase its degree each time a new node enters the network.
  - A new node enters with m links. Hence, node i has m chances to be chosen.
  - Let N be the number of nodes at time t, the instant of adding a new node. N = N(t) = N(t-1) + 1.
- Let k_i be a time-dependent continuous real variable (k_i is the degree of node i that enters the network at time t_i), the rate at which node i acquires links follows the equation:

$$\frac{\partial k_i}{\partial t} = m\Pi(k_i) = m\frac{k_i}{\sum_{j=1}^{N-1} k_j}$$

$$\boxed{\frac{\partial k_i}{\partial t} = \frac{k_i}{2t-1}}$$

The sum in the denominator goes over all nodes in the network, except the newly added node. A total of (mt) links are added by time t. The factor 2 is because a link is accounted twice, once for each of its end nodes.

$$\sum_{j=1}^{N-1} k_j = 2mt - m$$

Time Dependent Degree of a Node

- For larger t, the term (-1) can be neglected in the denominator, obtaining:

$$\frac{\partial k_i}{k_i} = \frac{1}{2}\frac{\partial t}{t}$$

- Integrating the above equation with the fact that $k_i(t_i) = m$, meaning that node i joins the network at time t_i with *m* links, we obtain:

For any of the existing node *i* at time *t*,

$$k_i(t) = m\left(\frac{t}{t_i}\right)^{\beta}$$

where β = ½ is called the network's dynamical exponent.

Observations:

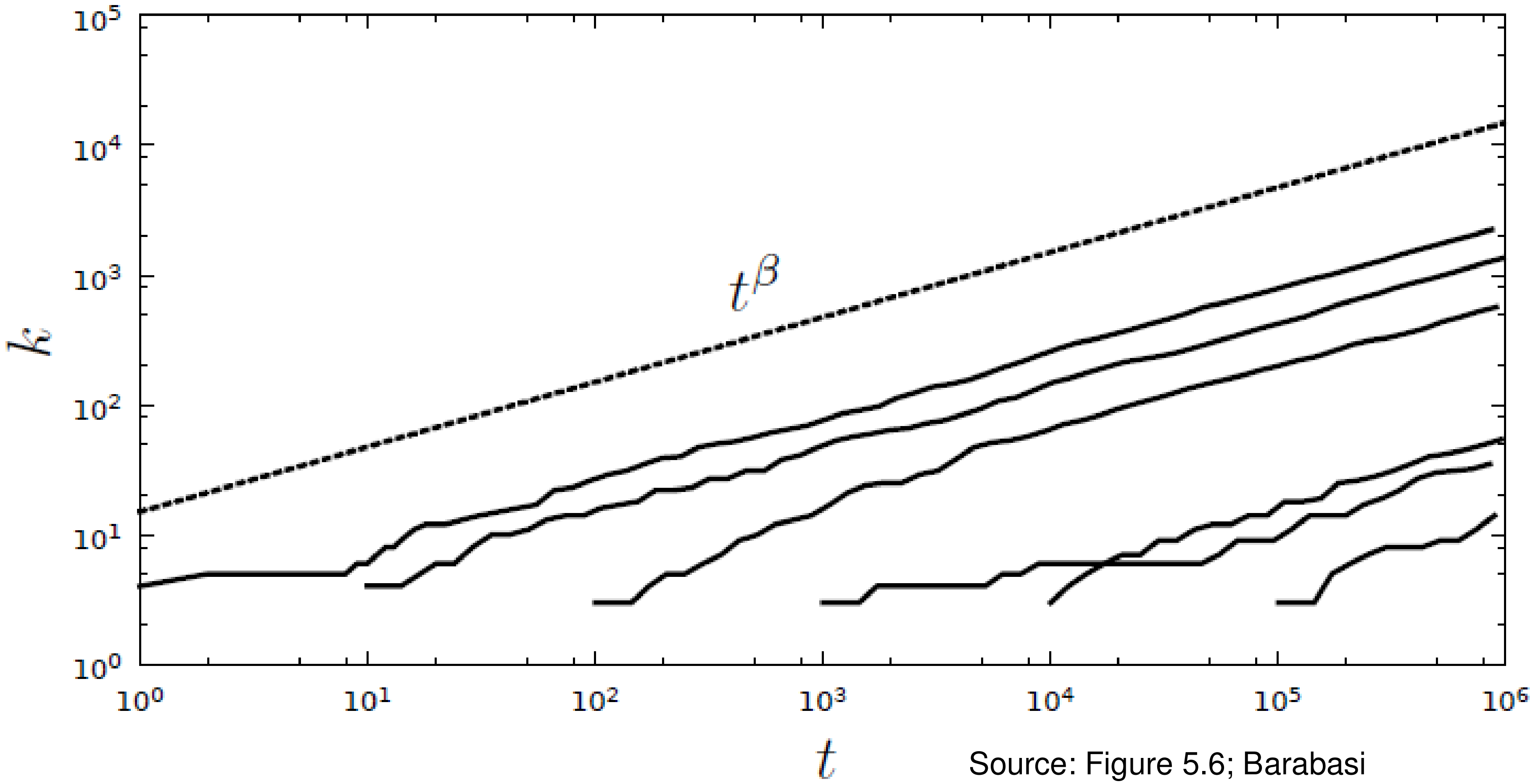
1) The degree of each node increases following the above power law.
2) Each new node has more nodes to link than the previous nodes. In other words, with time, each node competes for links with an increasing pool of nodes.

Time Dependent Degree of a Node

- The earlier node *i* was added, the higher is its degree $k_i(t)$.
  - Hence, hubs are large not because they grow faster, but because they arrived earlier.
  - The growth in the degrees is sub linear ($\beta < 1$).
- The rate at which node i acquires new links is given by the derivative: $$\frac{dk_i(t)}{dt} = \frac{m}{2}\frac{1}{\sqrt{t_i t}}$$
- Indicating that older nodes acquire more links in a unit time (as they have smaller t_i), as well as that the rate at which a node acquires links decreases with time as $t^{-1/2}$. Hence, less and less links go to a node with time.
- Thus, the BA model offers a dynamical description of a network's evolution: in real networks, nodes arrive one after the other, connecting to the earlier nodes.
  - This sets up a competition for links during which the older nodes have an advantage over the younger nodes, eventually turning into hubs.

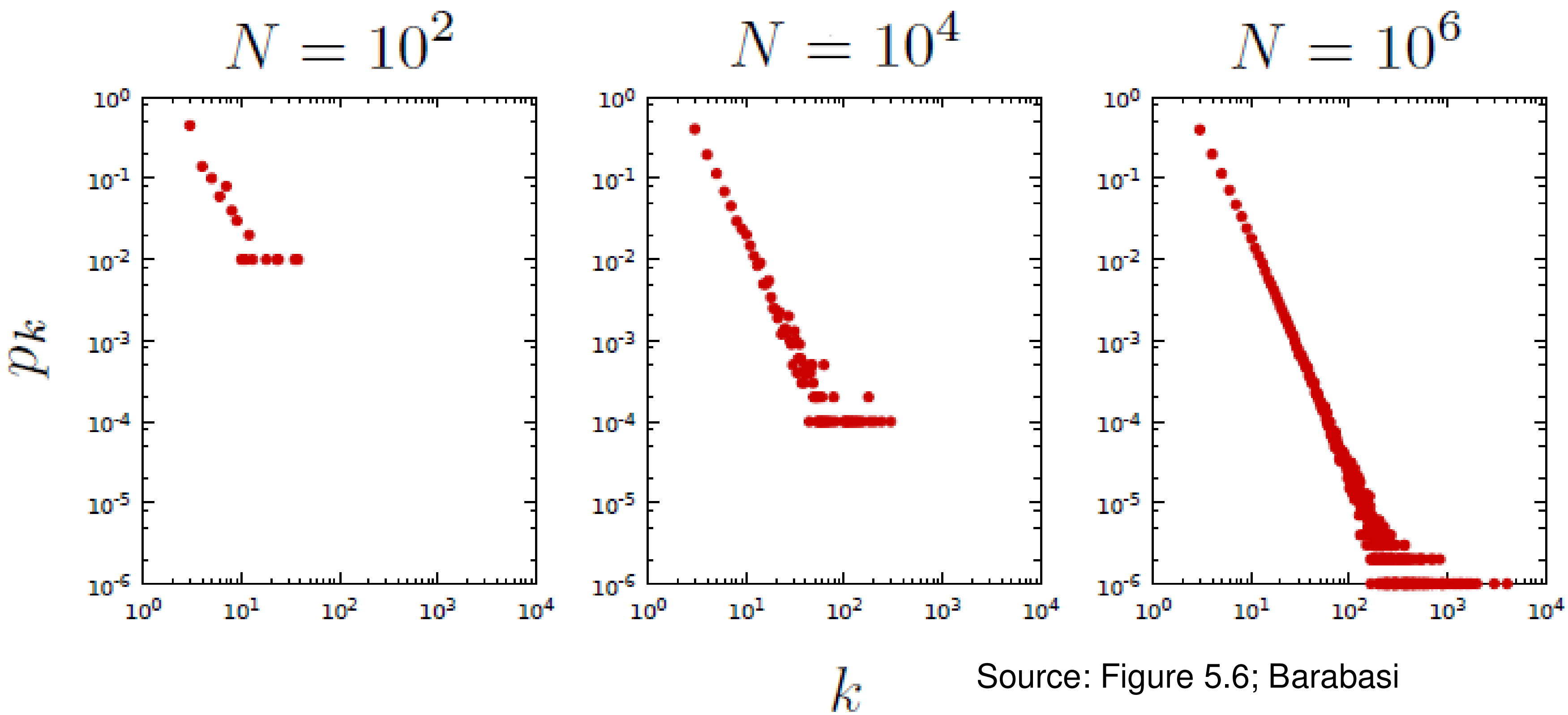
Degree Dynamics

We plot here the time dependence of the degrees of nodes added at time t = 1, 10, 10^2, 10^3, 10^4, 10^5. The degree of each of these nodes increases according to the t^β law (analytical prediction plotted in dotted lines for node added at time 1.



Source: Figure 5.6; Barabasi

Degree Dynamics



Source: Figure 5.6; Barabasi

Degree distribution of a network after the addition of N = 10^2, 10^4 and 10^6 nodes according to the power law at time t = 10^2, 10^4 and 10^6.
The larger the network, the more obvious is the power-law nature of the degree distribution.

Bianconi-Barabasi (BB) Model Motivation

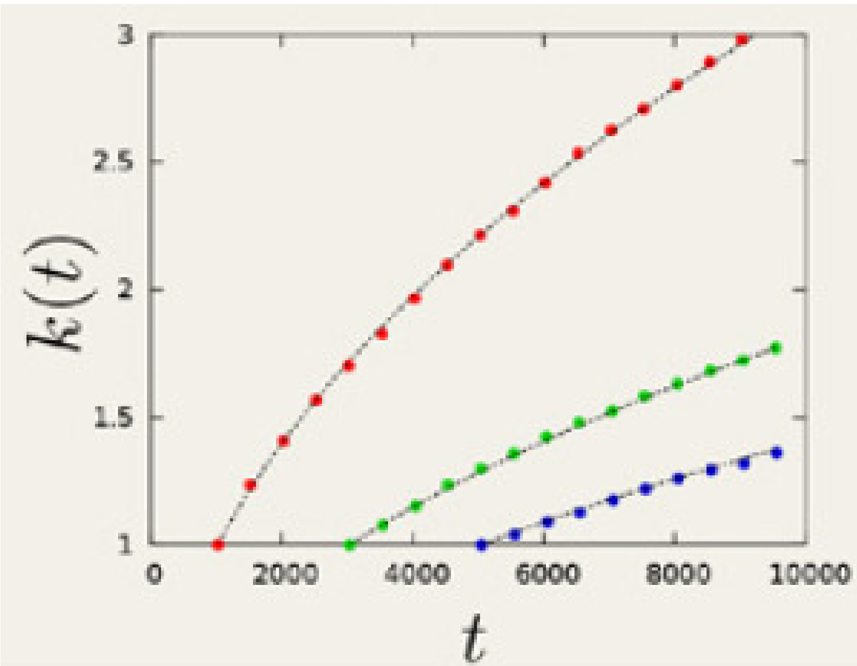
- The Barabasi-Albert model leads to a scenario where the late nodes can never turn into the largest hubs.
- In reality, a node's growth does not depend on the node's age only.
  - Instead web pages, companies or actors have intrinsic qualities that influence the rate at which they acquire links.
    - Some show up late and nevertheless grab most links within a short timeframe.
    - Example: Though, Facebook came later than Google, Facebook is the most linked node in the Internet.
- The goal of this model is to understand how the differences in the node's ability to acquire links, and other processes not captured by the Barabasi-Albert model, like node and link deletion or aging, affect the network topology.

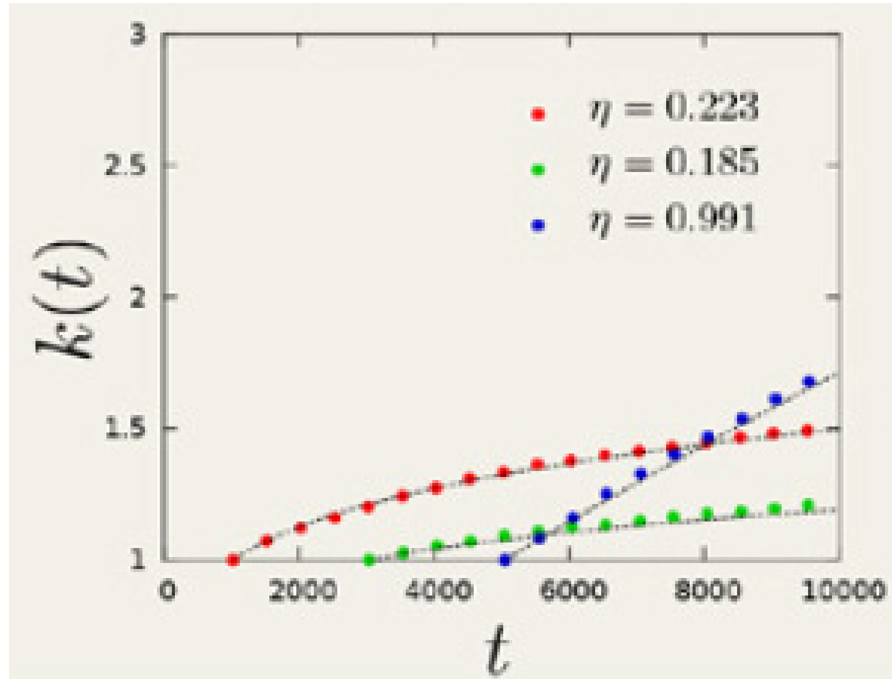
Bianconi-Barabasi (BB) Model

- Fitness – the intrinsic property of a node that propels more nodes towards it.
- The Barabasi-Albert model assumed that a node's growth rate is determined solely by its degree.
- The BB model incorporates the role of fitness and assumes that preferential attachment is driven by the product of a node's fitness, η, and its degree k.
- Growth: In each timestep, a new node j with m links and fitness η_j is added to the system, where η_j is a random number chosen from a distribution ρ(η) [for example: uniform distribution].
  - Once assigned, a node's fitness does not change.
- Preferential Attachment: The probability that a link of a new node connects to a pre-existing node i is proportional to the product of node i's degree k_i and its fitness η_i.

$$\Pi_i = \frac{\eta_i k_i}{\sum_j \eta_j k_j}$$

BA Model vs. BB Model





BB Model

$$\frac{\partial k_i}{\partial t} = m \frac{\eta_i k_i}{\sum_k \eta_j k_j}$$

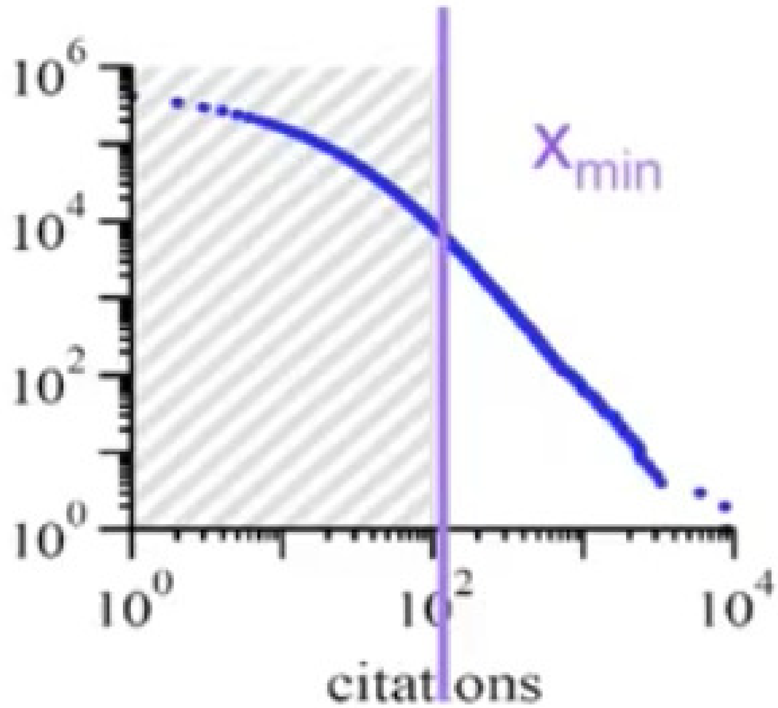
$$k_{\eta_i}(t, t_i) = m \left(\frac{t}{t_i} \right)^{\beta(\eta_i)}$$

where $\beta(\eta_i)$ is a fitness-dependent dynamic exponent of node i.

A node with a higher fitness will increase its degree faster.

Where does the Power-Law distribution start for real networks?

- If $P(x) = C X^{-\alpha}$, then Xmin needs to be certainly greater than 0, because $X^{-\alpha}$ is infinite at $X = 0$.
- Some real-world distributions exhibit power-law only from a minimum value (x_{min}).



Source:MEJ Newman,
Power laws, Pareto distributions and Zipf's law, Contemporary Physics 46, 323–351 (2005)

Some Power-Law Exponents of Real-World Data

	x_{min}	exponent α
frequency of use of words	1	2.20
number of citations to papers	100	3.04
number of hits on web sites	1	2.40
copies of books sold in the US	2 000 000	3.51
telephone calls received	10	2.22
magnitude of earthquakes	3.8	3.04
diameter of moon craters	0.01	3.14
intensity of solar flares	200	1.83
intensity of wars	3	1.80
net worth of Americans	$600m	2.09
frequency of family names	10 000	1.94
population of US cities	40 000	2.30

Not every network is power law distributed

- There is a limit to how many relationships a node can maintain
- Frequent email communication
  - Taking into consideration whether there is a sequence of email exchanges between people
- Thesaurus – a word is not the synonym for every other word out there.
- Network of directors – A person cannot be in several director boards with every other person out there.