CSC 641 Network Science, Fall 2015

Instructor: Dr. Natarajan Meghanathan

Take Home Exam 3

Due: Wednesday Dec. 2, 2015: 6 PM

Late submission (Dec. 2, 2015-6:10 PM to Dec. 3, 2015-6 PM: -30 points, taken off from your score)
Late submission (Dec. 3, 2015-6:01 PM to Dec. 4, 2015-6 PM: -60 points, taken off from your score)
No late submission allowed after Dec. 4, 2015-6 PM.

Maximum Points: 100
Q1, Q7, Q8 - 15 points each Q2, Q3, Q4, Q5, Q6 - 11 points each

Q1) a) Consider the BA model for scale-free networks wherein a new node joins the network for every time unit (t_1, t_2, t_3, ...) and the id of a node is simply the index of the time unit (1, 2, 3, ...) the node joins the network. Each new node joining the network connects to m of the existing nodes according to preferential attachment based on node degree.

At some time t, given the rate of change of the degree of node i is: $\frac{\partial k_i(t)}{\partial t} = \frac{k_i(t)}{2t-1}$, derive a closed-form expression for the degree of node i as a function of t, $k_i(t)$. Show all the steps of your integration.

b) At some time unit t, if the degree of a node that joined the network at time X units is Y, compute the degree of the node that joined the network at time Z units.

	X	Y	Z
Karthik	10	50	100
Anirudh	5	25	75
Yashwanth	15	30	40

Q2) Consider a scale-free network modeled according to the power-law distribution $p(k) = Ck^{-\gamma}$. Let the power-law exponent (γ) be as assigned to you. The minimum possible degree for any node in the network is kmin. For such a network, **determine a numerical value for the probability of finding a node with degree** k.

	γ	kmin	k
Karthik	2.5	2	3
Anirudh	2.2	3	5
Yashwanth	2.8	1	4

Q3) Consider a network modeled using the power law, $p(k) = k^{-\gamma}$. **Determine the value of the power-law exponent γ** if the network has approximately X% of nodes with degree k.

	X%	k
Karthik	4%	4
Anirudh	3%	3
Yashwanth	5%	5

Q4) Consider a scale-free network of N = 1000 nodes modeled using the power-law, $p(k) = Ck^{-\gamma}$. The minimum and maximum degrees of the nodes in the network are kmin = 1 and kmax = 10 respectively. **Find the power-law exponent (γ) and the constant C**.

	N	kmin	kmax
Karthik	1000	3	20
Anirudh	100	1	10
Yashwanth	500	2	15

Q5) Consider the enhanced WS model for small-world networks. Let there be a regular graph that is transformed to a small-world network. For every edge (*u*, *v*) selected for re-wiring, the probability that a node *w* of distance d(u, w) hops to *u* is picked for re-wiring is p(w) and the probability that a node *w'* of distance d(u, w') hops to *u* is picked for re-wiring is p(w'). Find the value for the **parameter *q*** in the enhanced WS model.

	d(u, w)	p(w)	d(u, w')	p(w')
Karthik	2	0.2	4	0.08
Anirudh	3	0.15	5	0.05
Yashwanth	4	0.10	7	0.02

Q6) Consider a regular ring lattice of degree kregular for every node. This regular graph is transformed to a small-world network by arbitrarily re-wiring the edges with probability β. Let the clustering coefficient of the small-world network generated out of this re-wiring be C(β). **Determine the re-wiring probability β**.

	kregular	C(β)
Karthik	4	0.04
Anirudh	6	0.03
Yashwanth	8	0.02

Q7) Consider the BB model for scale-free networks .
Let the parameter $\beta(\eta_i)$ for any node *i* be equal to the fitness of node *i*, η_i. Consider two nodes A and B such that the fitness of node B is twice the fitness of node A.
Node A joins the network at time 10 units and node B joins the network at time 100 units.
If the degree of the nodes increase for every time unit (when a new node joins), **what is the *minimum* value of the time unit starting from which the degree of node B would always be greater than the degree of node A**? Show all the steps. No guess work.

Q8) For a probability distribution p(k), consider the first moment (mean) to be given by: $\int_1^{\infty} kp(k)dk$, and the second moment is given by: $\int_1^{\infty} k^2 p(k)dk$. For the power-law distribution $p(k) = k^{-\gamma}$, **find the minimum value of the power-law exponent γ that the first moment is defined (i.e., positive) and similarly, find the minimum value of γ that the second moment is defined (i.e., positive)**. Show all the steps of your integration. No guess work