Student Name: _____ J#: _____

CSC 323 Algorithm Design and Analysis, Spring 2016

Instructor: Dr. Natarajan Meghanathan

Quiz 1 (February 2, 2016) Max. Points: 25 Max. Time: 15 min.

1) (9 pts) Solve the summation below:

$$\sum_{j=0}^{n-2}\sum_{k=0}^{n-j-2} 1 = \sum_{j=0}^{n-2}(n-j-2-0+1) = \sum_{j=0}^{n-2}(n-1-j)$$

$$= (n-1) + (n-2) + \ldots + (1) = \frac{n(n-1)}{2}$$

($j=0$, $j=1$, $j=n-2$)

2) (8 points) Find the class O(g(*n*)) for the function: $\sqrt{2n^3+5n^2+3n+2}$

Use reverse side for additional space, if needed

Let $g(n) = n^2 = \sqrt{n^4}$

$$\lim_{n\to\infty} \frac{\sqrt{2n^3+5n^2+3n+2}}{g(n)=n^2} = \lim_{n\to\infty} \frac{\sqrt{2n^3+5n^2+3n+2}}{\sqrt{n^4}}$$

$$= \lim_{n\to\infty} \sqrt{\frac{2n^3+5n^2+3n+2}{n^4}} = \lim_{n\to\infty} \sqrt{\frac{2}{n}+\frac{5}{n^2}+\frac{3}{n^3}+\frac{2}{n^4}} = 0.$$

Hence, $\sqrt{2n^3+5n^2+3n+2} = O(n^2)$

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3) (8 points) Derive the asymptotic relationship between the two functions: $n^2\log(n)$ and $n\log(n^{100})$

Note that $n \cdot \log n^{100} = 100 \times n \times \log n$.

$$\lim_{n\to\infty} \frac{n^2 \log n}{n \log n^{100}} = \lim_{n\to\infty} \frac{n^{\cancel{2}} \cancel{\log n}}{100 \cancel{n} \cancel{\log n}} = \lim_{n\to\infty} \frac{n}{100} = \infty$$

Hence, $n^2 \log n = \Omega(n \log n^{100})$

$\Rightarrow$ $n \log n^{100} = O(n^2 \log n)$