Student Name: ______________________________ J#: ________________

CSC 323 Algorithm Design and Analysis, Spring 2016

Instructor: Dr. Natarajan Meghanathan

Quiz 2 (February 9, 2016) Max. Points: 25 Max. Time: 15 min.

1) (9 pts) The following algorithm returns the index of the array if the value of the element at that index equals the index itself. If there is no such entry in the array, the algorithm returns -1.

Input: Array A with indices 0 to n-1
Output: Index *i* if A[i] = i or -1
Algorithm

```
for i = 0 to n-1 do
    if (A[i] = i) then
        return i
end for
return -1
```

Determine the overall time-complexity of the algorithm with respect to an appropriate asymptotic notation (O or Θ). Show all the work.

Best-case: 1 comparison

Worst-case: $\sum_{i=0}^{n-1} 1 = (n-1) - 0 + 1 = n$ comparisons

Overall: $\lim_{n \to \infty} \frac{\text{Best-case}}{\text{Worst-case}} = \lim_{n \to \infty} \frac{1}{n} = 0$

$= \underline{O(n)}$

2) (7 pts) Solve the recurrence relation: M(n) = M(n-1) + 3 for n > 1; M(1) = 0

Use reverse side for additional space, if needed

$M(n) = M(n-1) + 3$ for $n > 1$

$M(n-1) = M(n-2) + 3$

$M(n) = M(n-2) + 3 + 3 = M(n-2) + 2*3$

$M(n-2) = M(n-3) + 3$

$M(n) = M(n-3) + 3 + 2*3 = M(n-3) + 3*3$

$M(n) = M(n-i) + i*3$

Let $i = n-1$

$M(n) = M(n-(n-1)) + (n-1)*3 = M(1) + 3(n-1) = 3(n-1)$

$= \underline{\Theta(n)}$

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3) (9 pts) Solve the recurrence relation: M(n) = M(n/5) + 1 for n > 1; M(1) = 0.

$M(n) = M(n/5) + 1$ for $n > 1$

Let $n = 5^k$ $M(1) = M(5^0) = 0$ $(k=0)$

$M(5^k) = M(5^{k-1}) + 1$ for $k > 0$.

$n > 1$
$5^k > 1$
$k > 0$

$M(5^{k-1}) = M(5^{k-2}) + 1$

$M(5^k) = M(5^{k-2}) + 1 + 1 = M(5^{k-2}) + 2$

$M(5^{k-2}) = M(5^{k-3}) + 1$

$M(5^k) = M(5^{k-3}) + 1 + 2 = M(5^{k-3}) + 3$

$M(5^k) = M(5^{k-i}) + i$

let $i = k$

$M(5^k) = M(5^0) + k = 0 + k = k$

$n = 5^k \Rightarrow k = \log_5 n$

$M(n) = \log_5 n = \Theta(\log n)$