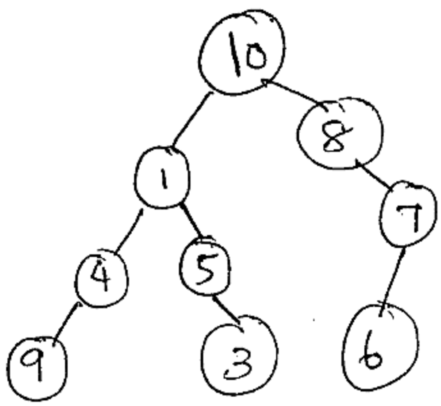
Student Name: ______________________________ J#: ________________

CSC 323 Algorithm Design and Analysis, Spring 2016
Instructor: Dr. Natarajan Meghanathan

Quiz 4 (March 1, 2016) Max. Points: 25 Max. Time: 15 min.

1) (8 pts) Draw a binary tree with 9 nodes labeled: 1, 3, 4, 5, 6, 7, 8, 9, 10 in such a way that the in-order and pre-order traversals of the tree yield the following lists:
9, 4, 1, 5, 3, 10, 8, 6, 7 (in-order)
10, 1, 4, 9, 5, 3, 8, 7, 6 (pre-order)

Also, determine a post-order listing of the vertices of the binary tree.



Post-order 9, 4, 3, 5, 1, 6, 7, 8, 10.

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2) (8 pts) Determine whether 75 and 32 are relatively prime using the Euclid's GCD formula. Compare the number of divisions encountered with that of a brute-force approach.

~~GCD(75, 32) = GCD(32, 11) = GCD(11, 10) = GCD(10, 1)~~

$GCD(75, 32) = GCD(32, 75 \bmod 32) = GCD(32, 11)$ ①

$GCD(32, 11) = GCD(11, 32 \bmod 11) = GCD(11, 10)$ ②

$GCD(11, 10) = GCD(10, 11 \bmod 10) = GCD(10, 1) = 1$ ③

75 and 32 are relatively prime
3 divisions with GCD formula | 31 divisions brute force approach

3) (9 pts) Given the graph below, determine the number of paths of length 4 between vertices 'b' and 'c' using the adjacency matrix multiplication approach.

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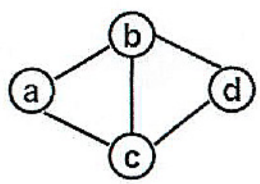

$$A = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

$$A^2 = A \times A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 1 & 2 \\ 1 & 3 & 2 & 1 \\ 1 & 2 & 3 & 1 \\ 2 & 1 & 1 & 2 \end{bmatrix}$$

$$A^4 = A^2 \times A^2 = \begin{array}{c} a \\ b \\ c \\ d \end{array} \begin{bmatrix} 2 & 1 & 1 & 2 \\ \boxed{1 \quad 3 \quad 2 \quad 1} \\ 1 & 2 & 3 & 1 \\ 2 & 1 & 1 & 2 \end{bmatrix} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 2 & 1 & 1 & 2 \\ 1 & 3 & 2 & 1 \\ 1 & 2 & 3 & 1 \\ 2 & 1 & 1 & 2 \end{bmatrix} \end{array}$$

paths of length 4 between 'b' and 'c' is
$1 \times 1 + 3 \times 2 + 2 \times 3 + 1 \times 1 = \underline{14}$