Module 3
Greedy Strategy

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Introduction to Greedy Technique

- Main Idea: In each step, choose the best alternative available in the hope that a sequence of locally optimal choices will yield a (globally) optimal solution to the entire problem.
- Example 1: Decimal to binary representation (objective: minimal number of 1s in the binary representation): Technique – Choose the largest exponent of 2 that is less than or equal to the unaccounted portion of the decimal integer.

	64	**32**	**16**	**8**	**4**	**2**	**1**
To represent 75:	1	0	0	1	0	1	1

- Example 2: Coin Denomination in US – Quarter (25 cents), Dime (10 cents), Nickel (5 cents) and Penny (1 cent).
- Objective: Find the minimum number of coins for a change
- Strategy: Choose the coin with the largest denomination that is less than or equal to the unaccounted portion of the change.
- For example, to find a change for 48, we would choose 1 quarter, 2 dimes and 3 pennies. The optimal solution is thus 6 coins and there cannot be anything less than 6 coins for US coin denominations.

Greedy Technique: Be careful!!!

- Greedy technique (though may appear to be computationally simple) cannot always guarantee to yield the optimal solution. It may end up only as an approximate solution to an optimization problem.
- For example, consider a more generic coin denomination scenario where the coins are valued 25, 10 and 1. To make a change for 30, we would end up using 6 coins (1 coin of value 25 and 5 coins of value 1 each) following the greedy technique. On the other hand, if we had used a dynamic programming algorithm for this generic version, we would have end up with 3 coins, each of value 10.

Fractional Knapsack Problem (Greedy Algorithm): Example 1

Knapsack weight is 6lb.

Item	1	2	3	4	5
Value, $	25	20	15	40	50
Weight, lb	3	2	1	4	5
Value/Weight	8.3	10	15	10	10

- **Greedy Strategy:** Pick the items in the decreasing order of the Value/Weight.
- Break the tie among the items the same Value/Weight by picking the item with the lowest Item index
- An optimal solution would be:
- Item 3 (1 lb), Item 2 (2 lb), and 3 lbs of Item 4.
- The maximum total Value of the items would be: $65
- Item 3 ($15), Item 2 ($20) and Item 4((3/4)*40 = $30)

- **Dynamic Programming:** If the items cannot be divided, and we have to pick only either the full item or just leave it, then the problem is referred to as an Integer (a.k.a. 0-1) Knapsack problem, and we will look at it in the module on Dynamic Programming.

Fractional Knapsack Problem (Greedy Algorithm): Example 2

Knapsack weight = 5 lb.

Item	1	2	3	4
Value, $	12	10	20	15
Weight, lb	2	1	3	2

Solution: Compute the Value/Weight for each item

Item	1	2	3	4
Value/Weight	6	10	6.67	7.5

Re-ordering the items according to the decreasing order of Value/Weight (break the tie by picking the item with the lowest Index)

Item	2	4	3	1
Value/Weight	10	7.5	6.67	6
Value, $	10	15	20	12
Weight, lb	1	2	3	2
Weight collected	1	2	2	

Items collected: Item 2 (1 lb, $10); Item 4 (2 lb, $15); Item 3 (2 lb, (2/3)*20 = $13.3);
Total Value = $38.3

Variable Length Prefix Encoding

- **Encoding Problem:** We want to encode a text that comprises of symbols from some n-symbol alphabet by assigning each symbol a sequence of bits called the codeword.
- If we assign bit sequences of the same length to each symbol, it is referred to as *fixed-length encoding*, we would need $\log_2 n$ bits per symbol of the alphabet and this is also the average # bits per symbol.
  - The 8-bit ASCII code assigns each of the 256 symbols a unique 8-bit binary code (whose integer values range from 0 to 255).
  - However, note that not all of these 256 symbols appear with the same frequency.
- **Motivation for Variable Code Assignment:** If we can come up with a code assignment such that symbols are assigned a bit sequence that is inversely related to the frequency of their occurrence (i.e., symbols that occur more frequently are given a shorter bit sequence and symbols that occur less frequently are given a longer bit sequence), then we could reduce the average number of bits per symbol.

- **Motivation for Prefix-free Code:** However, care should be taken such that if a given sequence of bits encoding a text is scanned (say from left to right), we should be able to clearly decode each symbol. In other words, we should be able to tell how many bits of an encoded text represent the i^{th} symbol in the text?

Huffman Codes: Prefix-free Coding

- **Prefix-free Code:** In a prefix-free code, no codeword is a prefix of a code of another symbol. With a prefix-free code based encoding, we can simply scan a bit string until we get the first group of bits that is a codeword for some symbol, replace these bits by this symbol, and repeat this operation until the bit string's end is reached.
- **Huffman Coding:**
  - Associate the alphabet's symbols with leaves of a binary tree in which all the left edges are labeled by 0 and all the right edges are labeled by 1.
  - The codeword of a symbol can be obtained by recording the labels on the simple path (a path without any cycle) from the root to the symbol's leaf.
- **Proof of correctness:** The binary codes are assigned based on a simple path traversed from the root to a leaf node representing the symbol. Since there cannot be a simple path from the root to a leaf node that leads to another leaf node (then we have to trace back some intermediate node – meaning a cycle). Hence, Huffman codes are prefix codes.

Huffman Algorithm

- **Assumptions:** The frequencies of symbol occurrence are independent and are known in advance.
- **Optimality:** Given the above assumption, Huffman's encoding yields a minimum-length encoding (i.e., the average number of bits per symbol is the minimum). This property of Huffman's encoding has lead to its use one of the most important file-compression methods.
  - Symbols that occur at a high-frequency have a smaller number of bits in the binary code, compared to symbols that occur at a low-frequency.

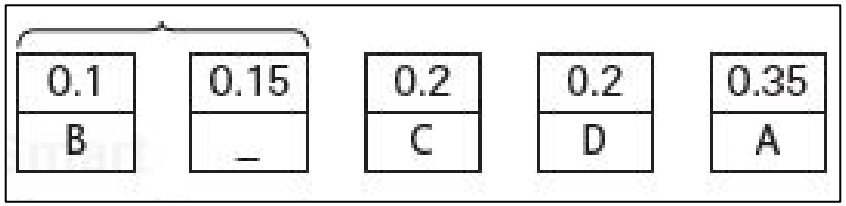
- **Step 1:** Initialize *n* one-node trees (one node for each symbol) and label them with the symbols of the given alphabet. Record the frequency of each symbol in its tree's root to indicate the tree's weight.
- **Step 2:** Repeat the following operation until a single tree is obtained:
  - Find two trees with the smallest weight (ties can be broken arbitrarily).
  - Make them the left and right sub trees of a new tree and record the sum of their weights in the root of the new tree as its weight.

Huffman Algorithm and Coding: Example

- Consider the five-symbol alphabet {A, B, C, D, -} with the following occurrence frequencies in a text made up of these symbols.
  - Construct a Huffman tree for this alphabet.
  - Determine the average number of bits per symbol.
  - Determine the compression ratio achieved compared to fixed-length encoding.

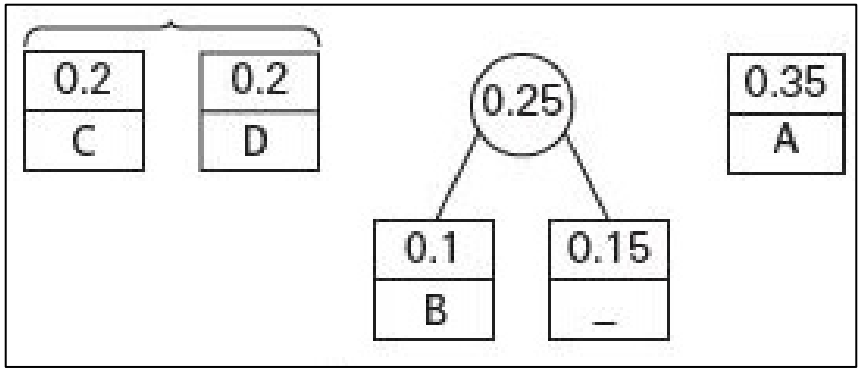
symbol	A	B	C	D	_
frequency	0.35	0.1	0.2	0.2	0.15

Initial



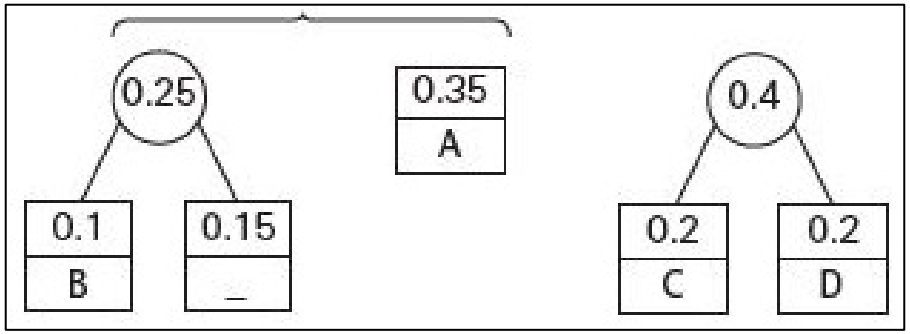
Break any tie by preferring to include the node with a smaller height to the right
If the height is not distinguishable, use Node ID, if possible; otherwise, break the ties arbitrarily.

Iteration - 1

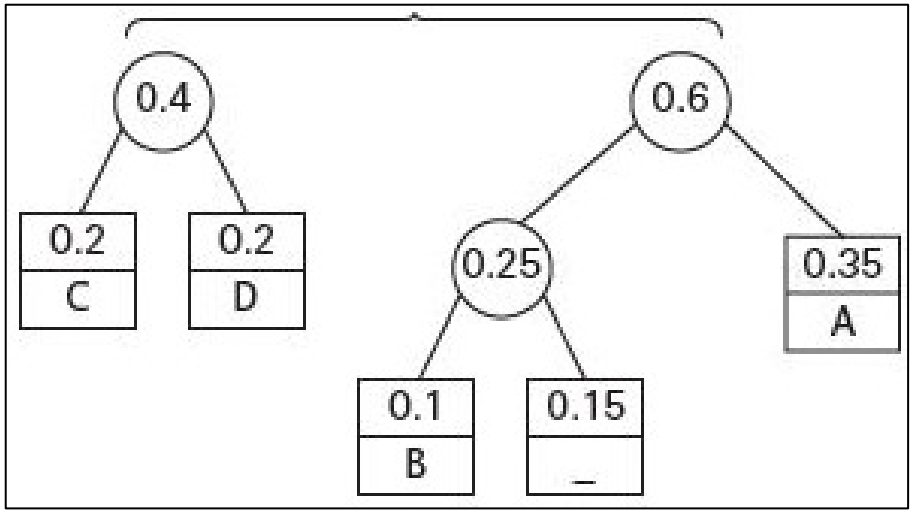


Huffman Algorithm and Coding: Example

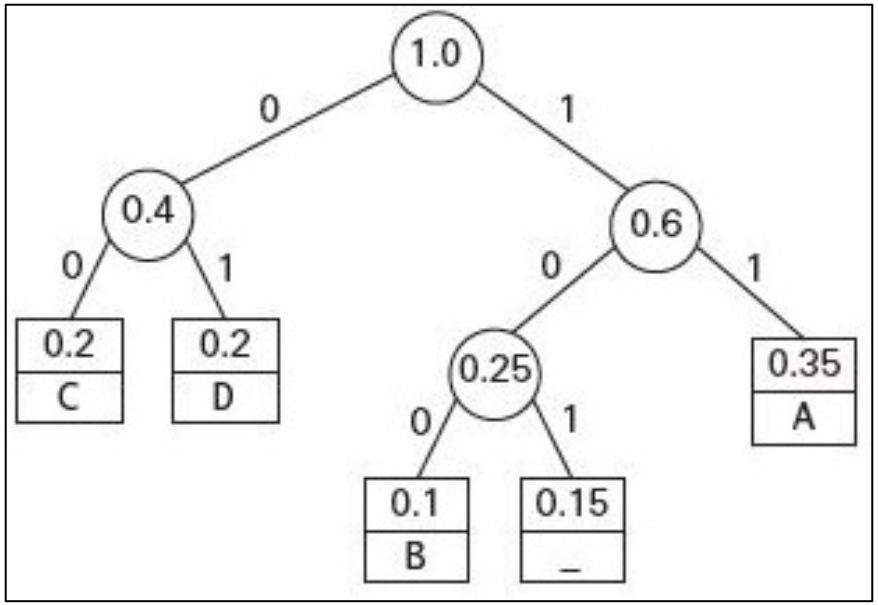
Iteration - 2



Iteration - 3



Iteration – 4 (Final)



symbol	A	B	C	D	_
frequency	0.35	0.1	0.2	0.2	0.15
codeword	11	100	00	01	101

Avg. # bits per symbol

= 2*0.35 + 3*0.1 + 2*0.2 + 2*0.2 + 3*0.15

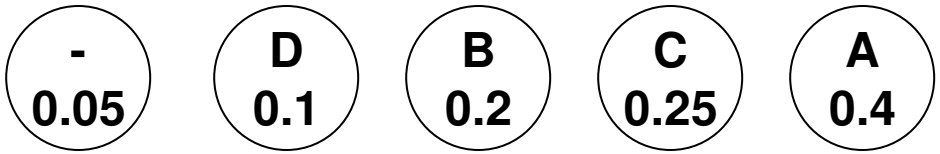
= 2.25 bits per symbol.

A fixed-length encoding of 5 symbols would require $\lceil \log_2 5 \rceil = 3$ symbols. Hence, the **Avg. Compression ratio** is 1 – (2.25/3) = 25%.

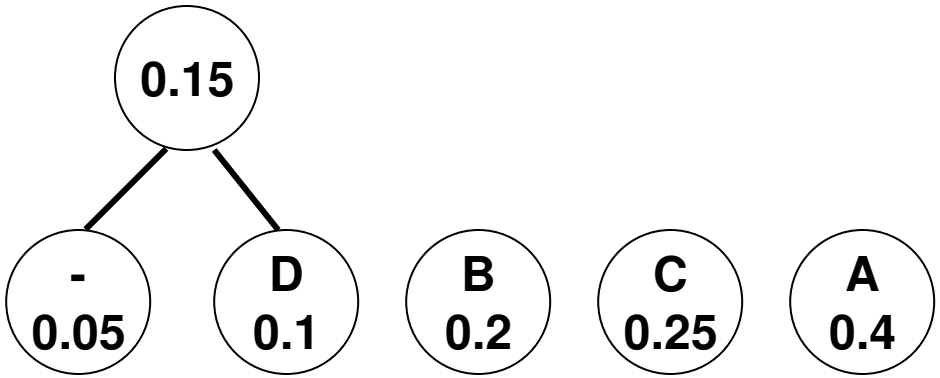
Huffman Coding: Example 2

A	0.4
B	0.2
C	0.25
D	0.1
-	0.05

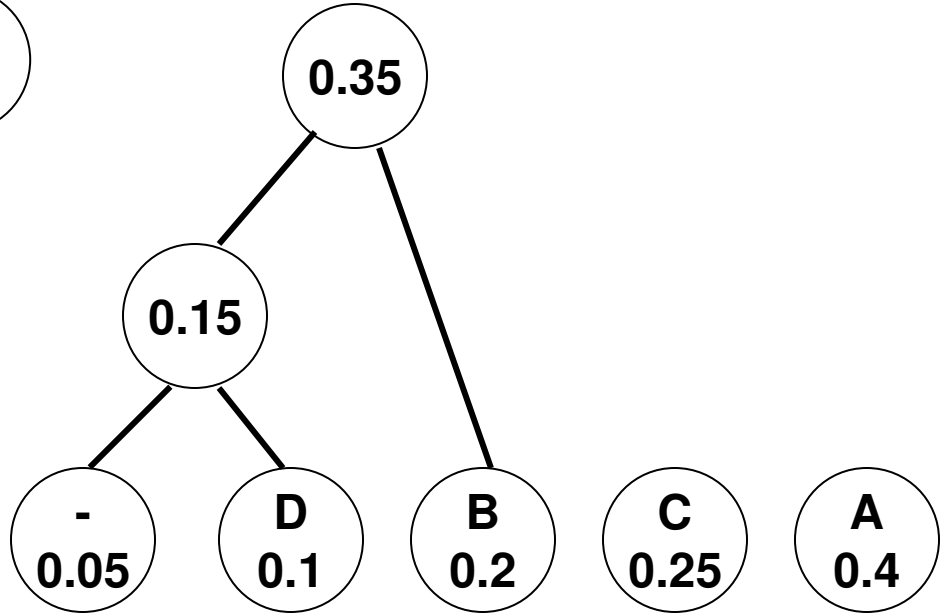
Initial



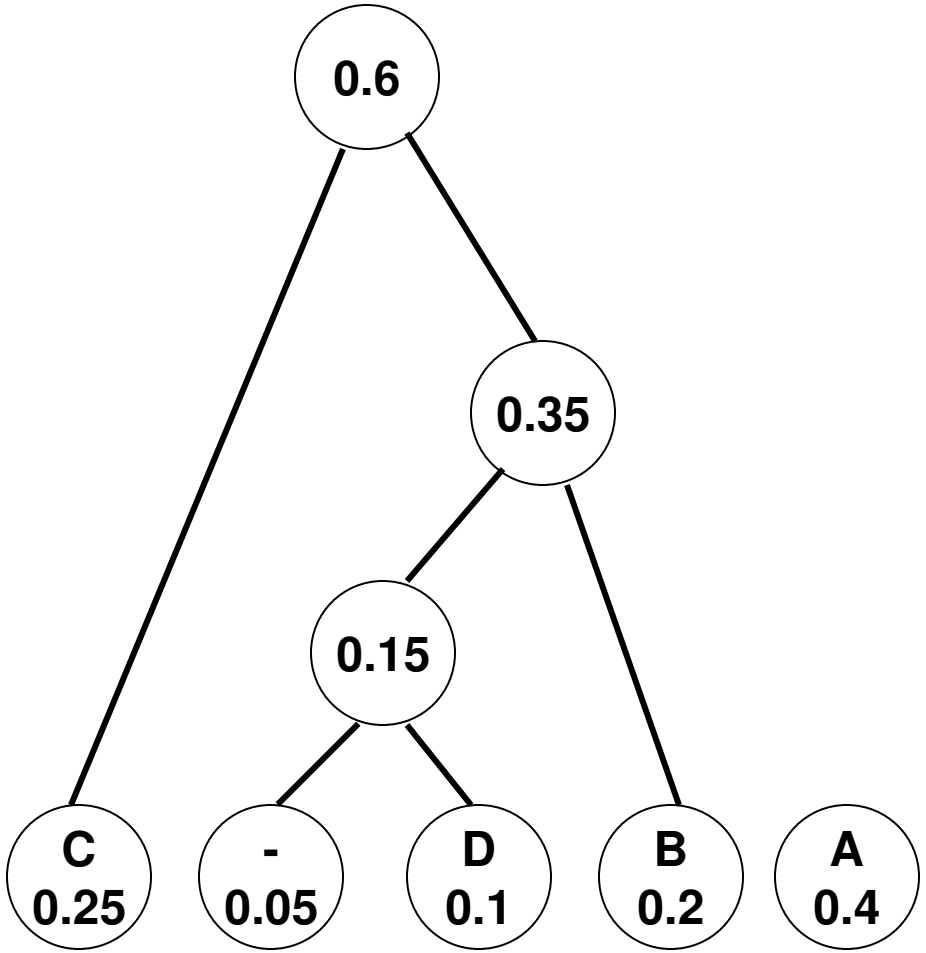
Iteration 1



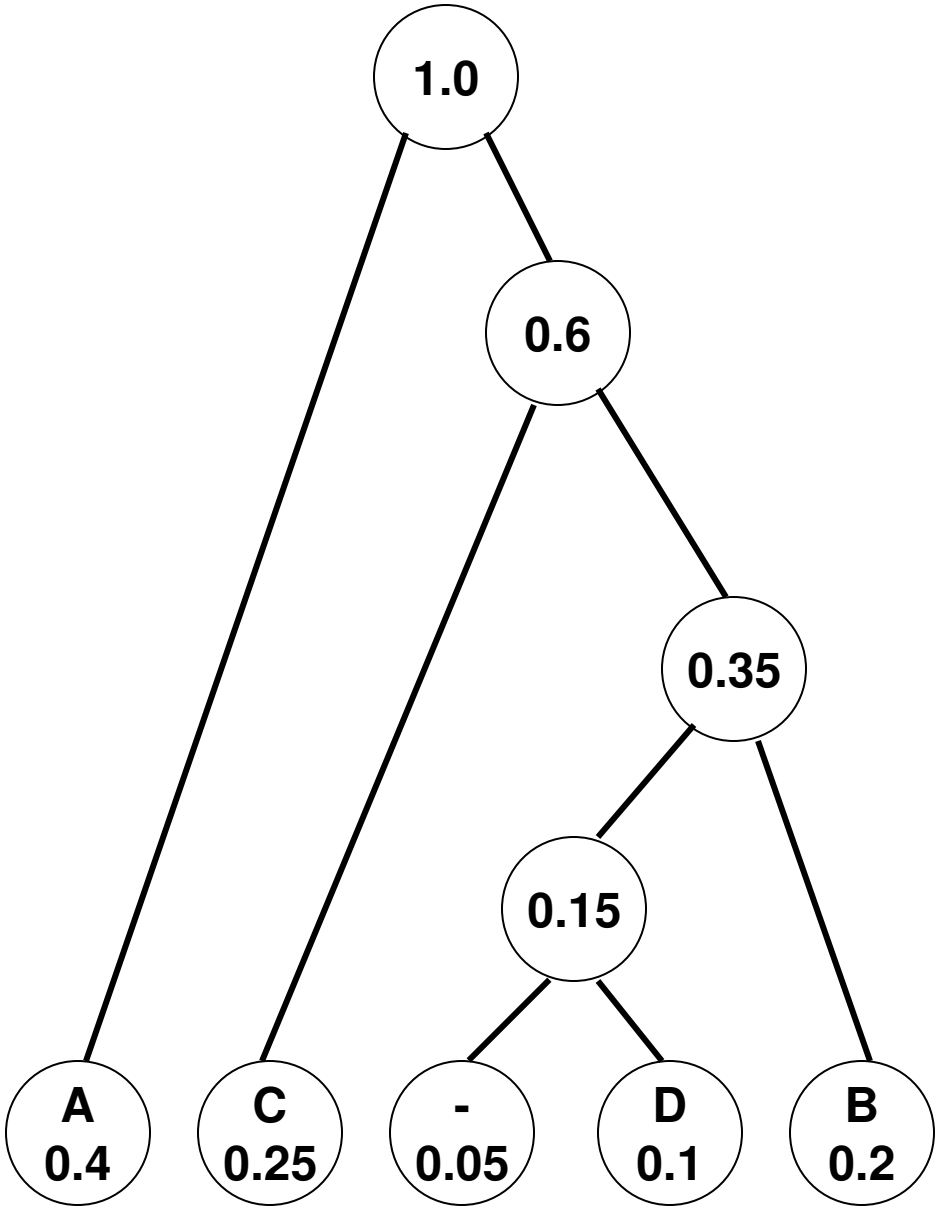
Iteration 2



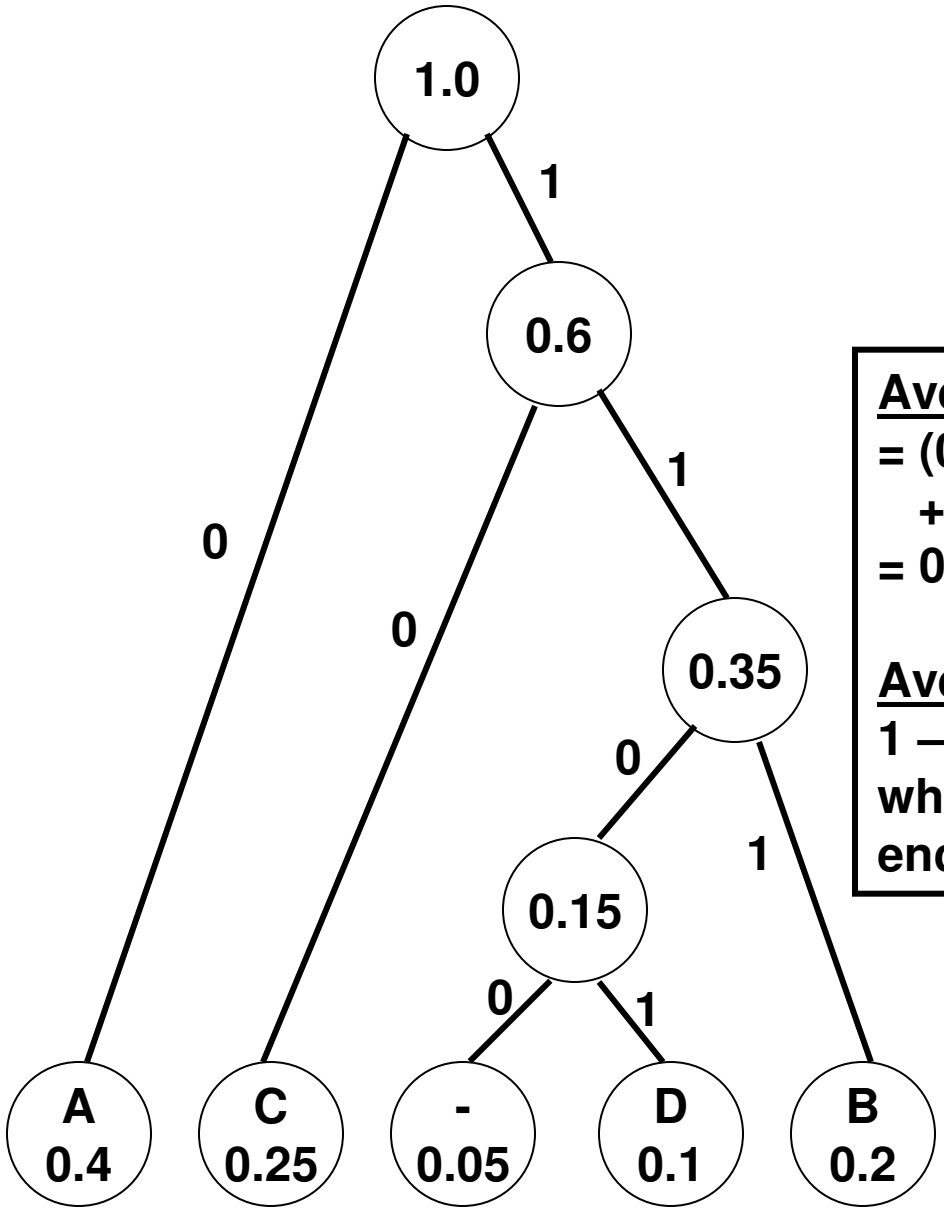
Iteration 3



Iteration 4



Huffman Tree



Huffman Codes

A	0.4	0
B	0.2	111
C	0.25	10
D	0.1	1101
-	0.05	1100

Average # bits per symbol (generic)

= (0.4)*(1) + (0.2)*(3) + (0.25)*(2) + (0.1)*(4) + (0.05)*(4)

= 0.4 + 0.6 + 0.5 + 0.4 + 0.2 = 2.1 bits/symbol

Average (Generic) Compression Ratio

1 – (2.1/3) = 0.3 = 30%

where 3 is the # bits/symbol under fixed encoding scheme.

Huffman Codes

A	**0.4**	**0**
B	**0.2**	**111**
C	**0.25**	**10**
D	**0.1**	**1101**
-	**0.05**	**1100**

Specific Character/Symbol Sequence:

A	A	B	C	A	C	D	–	A	B
0	0	111	10	0	10	1101	1100	0	111

Total # bits in the above sequence = 22 bits
Average # bits / symbol in the above sequence = 22 / 10 = 2.2 bits/symbol
where 10 is the number of symbols in the above sequence

If we had used fixed-length encoding, we would have used:
3 bits/symbol * 10 symbols = 30 bits

Compression ratio = 1 – (22/30) = 26.7%