

CSC 323 Algorithm Design and Analysis, Spring 2017

Instructor: Dr. Natarajan Meghanathan

Quiz 1 (January 31, 2017)

Max. Points: 30

Max. Time: 20 min.

1) (9 pts) Derive the asymptotic relationship between the two functions: $n^2 \log(n)$ and $n \log(n^{100})$

$$f(n) = n^2 \log n ; \quad g(n) = n \times \log(n^{100}) = 100n \times \log n$$

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \lim_{n \rightarrow \infty} \frac{n^2 \log n}{100n \log n} = \lim_{n \rightarrow \infty} \frac{n}{100} = \infty.$$

as $n \rightarrow \infty$, $f(n)$ grows faster than $g(n)$

$$\text{Hence, } g(n) = O(f(n))$$

$$\boxed{n \log(n^{100}) = O(n^2 \log(n))}$$

2) (9 pts) Let $f(n) = 5n^3 + 6n + 2$. Find a function $g(n)$ such that $f(n) = O(g(n))$ and $f(n) \neq \Theta(g(n))$. Show that your choice for $g(n)$ is correct using the Limits approach.

We need to choose a function $g(n)$ whose degree is strictly greater than 3, the degree of $f(n)$.

$$\text{Let } g(n) = n^4.$$

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \lim_{n \rightarrow \infty} \frac{5n^3 + 6n + 2}{n^4} = \lim_{n \rightarrow \infty} \frac{5}{n} + \frac{6}{n^3} + \frac{2}{n^4} = \underline{\underline{0}}$$

Hence, $g(n)$ grows much faster than $f(n)$ as $n \rightarrow \infty$.

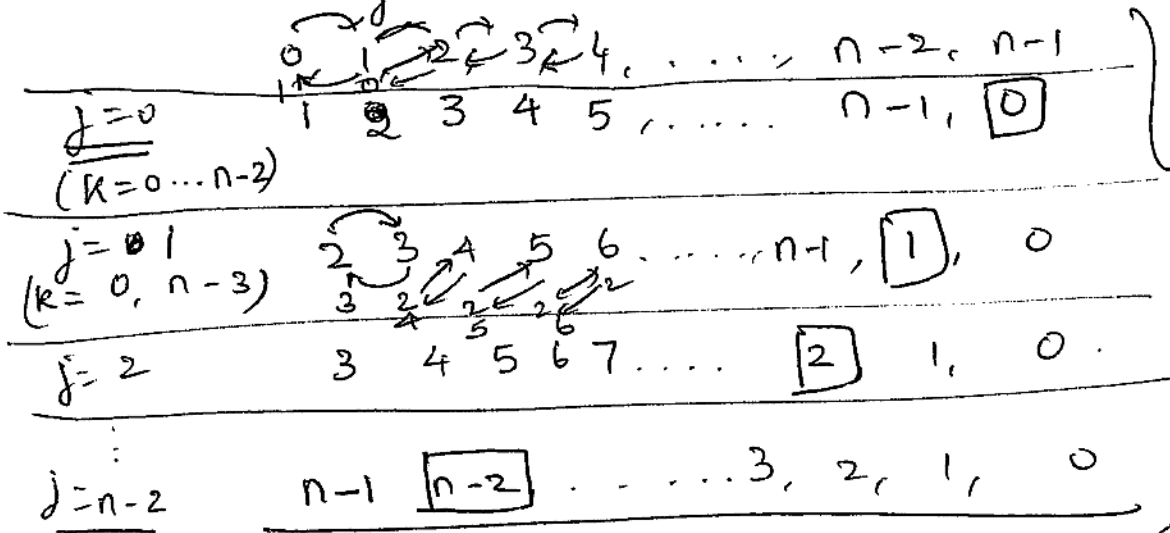
$$f(n) = O(g(n))$$

$$\underline{\underline{5n^3 + 6n + 2 = O(n^4)}}$$

3) (12 pts) Consider an array of n elements that needs to be sorted in the descending order. Given here is the code snippet of an algorithm for this purpose. It uses a Swap function to exchange arguments. If the input array to this code is $A[i] = i$, for $i = 0, 1, 2, \dots, n-1$, Show that the exact number of times the Swap function is called is $n(n-1)/2$.

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for (int j = 0; j ≤ n-2; j++)
    for (int k = 0; k ≤ n-j-2; k++)
        if (A[k] < A[k+1])
            Swap(A[k], A[k+1])
```

Given an array $A = 0, 1, 2, \dots, n-1$, we need to sort it in descending order $n-1, n-2, \dots, 2, 1, 0$.



We would need to execute the Swap function for each value of j and k .

times the swap function is executed is $\sum_{j=0}^{n-2} \sum_{k=0}^{n-j-2} \textcircled{1}$

$$= \sum_{j=0}^{n-2} [(n-j-2) - (0) + 1]$$

$$= \sum_{j=0}^{n-2} [n-j-1]$$

$$= (n-0-1) + (n-1-1) + (n-2-1) + \dots + (n-(n-2)-1)$$

$$= (n-1) + (n-2) + (n-3) + \dots + (1)$$

$$= \sum_{i=1}^{n-1} i = \frac{(n-1)(n-1+1)}{2} = \frac{n(n-1)}{2} = \underline{\underline{\Theta(n^2)}}$$