Student Name: _____ J#: _____

CSC 323 Algorithm Design and Analysis, Spring 2017

Instructor: Dr. Natarajan Meghanathan

Quiz 2 (February 7, 2017) Max. Points: 25 Max. Time: 15 min.

1) (10 pts) Solve the recurrence relation: M(n) = M(n-1) + 5 for n > 1; M(1) = 2

$M(n) = M(n-1) + 5$ for $n > 1$

$M(n-1) = M(n-2) + 5$

$M(n) = M(n-2) + 2*5$

$M(n-2) = M(n-3) + 5$

$M(n) = M(n-3) + 3*5$

$\vdots$

$M(n) = M(n-i) + i \times 5$ —— (general form)

Given $M(1) = 2$

put $n - i = 1$

i.e) $i = n - 1$ in the general form

$M(n) = M(n-(n-1)) + (n-1) \times 5$

$= M(1) + (n-1) \times 5$

$M(n) = 2 + (n-1) \times 5 = \underline{\Theta(n)}$

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2) (15 pts) Solve the recurrence relation: M(n) = M(n/5) + 1 for n > 1; M(1) = 2.

$M(n) = M(n/5) + 1$ for $n > 1$

$n > 1$
$5^k > 1$
$k > 0$

Let $n = 5^k$

$M(5^k) = M(5^{k-1}) + 1$ for $k > 0$
$M(5^0) = 2$

Let us solve the recurrence for k first

$M(5^k) = M(5^{k-1}) + 1$
$M(5^{k-1}) = M(5^{k-2}) + 1$
$M(5^k) = M(5^{k-2}) + 1 + 1 = M(5^{k-2}) + 2$
$M(5^{k-2}) = M(5^{k-3}) + 1$
$M(5^k) = M(5^{k-3}) + 1 + 2 = M(5^{k-3}) + 3$
⋮
$M(5^k) = M(5^{k-i}) + i$ ——— (general form)

Given $M(5^0) = 2$

put $k - i = 0$

i) put $i = k$ in the general form

$M(5^k) = M(5^{k-k}) + k = M(5^0) + k$

$M(5^k) = 2 + k$

replacing 5^k with n, and k with $\log_5^n$

$M(n) = 2 + \log_5^n = \underline{\Theta(\log n)}$