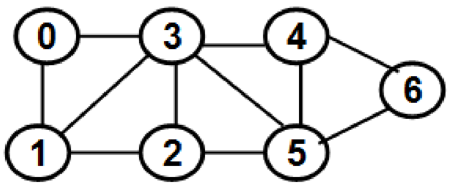
Random Networks: ER Model

1) Consider the real-world network graph given below.



(a) Determine the probability for a link (p_{link}) to exist between any two nodes if this real-world network graph were to be modeled as a random network graph.

(b) Using the p_{link} value determined in (a), generate a random network graph (that has the same number of nodes as the real-world graph given) according to the ER model.

(c) For the real-world network graph given, determine a distribution of the degree vs. average local clustering coefficient of the vertices with a particular degree: the K vs. Avg LCC(K) distribution.

(d) For the random network generated in (b), determine a distribution of the degree vs. average local clustering coefficient of the vertices with a particular degree: the K vs. Avg LCC(K) distribution.

(e) Determine the R^2 values for the distributions in (c) and (d). What can infer you from the values of the two R^2 values? Check Slide # 17 (newly added) in Module 5.

Scale Free Networks: Power Law Model

1) Consider a network modeled using the power-law, $P(K) = CK^{-\gamma}$. Determine the power-law exponent γ and the constant C if the network has approximately 15% of nodes with degree 4 and 5% of nodes with degree 6.

2) Generate a power-law distribution with degree exponent $\gamma = 2.45$. Determine the power-law constant and the average degree.

3) Given the following adjacency list for the vertices, Use the **Kurtosis** measure to determine whether the Degree distribution could be classified to exhibit "scale-free" property.

0	1
0	2
0	3
0	4
0	5
0	6
0	7
1	2
1	3
1	4
1	5

1	8
1	9
2	3
2	8
3	4
3	5
3	6
3	9
4	6
4	7
4	8
6	7
6	9

4) Given the following probability degree distributions (a) and (b):

K	P(K)
1	0.7780
2	0.1240
3	0.0423
4	0.0197
5	0.0109
6	0.0067
7	0.0045
8	0.0031

1) Draw a plot of the degree distribution and determine if the degree distribution follows a power-law or Poisson?
2) Determine the parameters of the degree distribution you decided as listed below.
   -- If the degree distribution follows Power Law, determine the Power Law Constant (C) and the Power Law Exponent (γ)
   -- If the degree distribution follows Poisson Law, determine the mean and standard deviation

5) Consider a scale-free network of N = 70 nodes modeled using the power-law, $P(K) = CK^{-\gamma}$. The minimum and maximum degrees of the nodes in the network are kmin = 2 and kmax = 35 respectively. Find the power-law exponent (γ), the power-law constant C and the average path length.

Scale Free Networks: BA and BB Models

1) Consider the following degree distribution of the nodes and their fitness.
- Determine the probability with which each node is likely to get the first link with a newly joining node under the BA and BB models.
- Let a new node join the network with 2 links under the BA and BB models. Determine which nodes are likely to get connected to the new node.

ID	Degree	Fitness
1	1	5
2	4	4
3	5	7
4	3	10
5	3	8
6	2	9
7	2	6

2) Consider the BB model for scale-free networks .
- Let the parameter $\beta(\eta i)$ for any node i be equal to the fitness of node i, ηi. Consider two nodes A and B such that the fitness of node B is <u>four times</u> the fitness of node A.

- Node A joins the network at time 20 units and node B joins the network at time 150 units.
- If the degree of the nodes increase for every time unit (when a new node joins), what is the *minimum* value of the time unit starting from which the degree of node B would always be greater than the degree of node A?

3) Consider the BB model for scale-free networks. Let the degree of a node A be 50 at time 100 units. If the fitness of node A is 2, compute the degree of node A at time 400 units.

4) At time 500 units, the following is the degree distribution of the nodes that joined at the time units indicated below. Determine the number of links added per node introduction (m) and the network's dynamical exponent (β). Estimate the degree of a node that joined the network at time 40 units.

Node joining Time, ti	Degree at Time t = 500
10	28
25	18
50	13
75	10
100	9
125	8
150	7

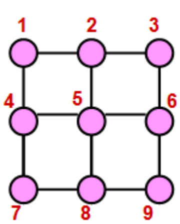
5) A node joined the network at time 20 units. Given below is the degree of the node at various time units. Determine the number of links added per node introduction and the fitness of the node. Under the BB model of evolution, assume the dynamical exponent value for a node is equal to the fitness of the node itself. Estimate the degree of the node at time 350 units.

Time Unit t	Degree at Time t
50	52
75	93
100	142
125	196
150	256
175	320
200	388

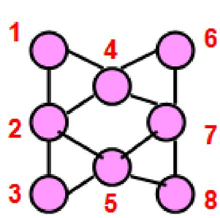
Small-World Networks

1) Consider a regular ring lattice of degree 12 for every node. This regular graph is transformed to a small-world network by arbitrarily re-wiring the edges with probability β. Let the clustering coefficient of the small-world network generated out of this re-wiring be 0.3. **Determine the re-wiring probability β**.

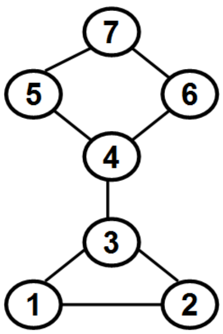
2) Consider each of the following networks that evolve to a small-world network under the "Enhanced Watts-Strogatz Model". For each network: let the link 1 - 2 be the first link chosen for rewiring. Predict the vertex to which vertex 1 will be rewired to in each case?



(a) q = 2



(b) q = 1



(c) q=1

3) Consider the enhanced WS model for small-world networks. Let there be a regular graph that is transformed to a small-world network. For every edge (u, v) selected for re-wiring, the probability that a node w of distance 3 hops to u is picked for re-wiring is 0.25 and the probability that a node w' of distance 5 hops to u is picked for re-wiring is 0.10. Find the value for the **parameter *q*** in the enhanced WS model.

Homophily

1) Consider the graph shown below. If nodes 1, 2, 3 and 4 are of type A and nodes 5, 6, 7, 8 are of type B, determine whether nodes of type A and B exist as one single community or two separate communities. Show all the work.