Module 8:
Heap

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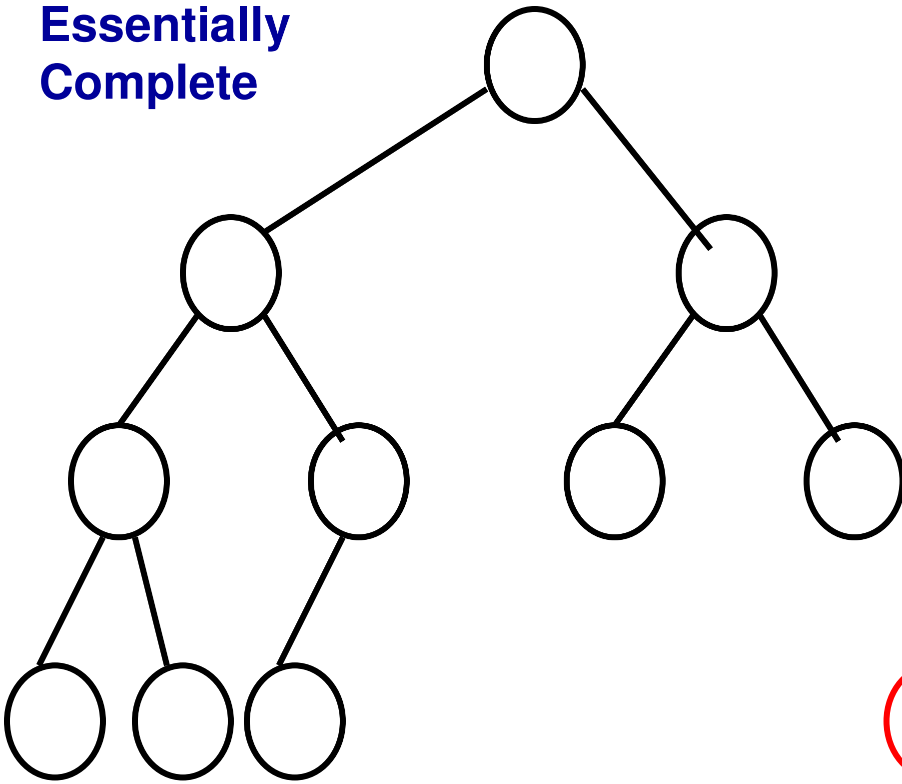
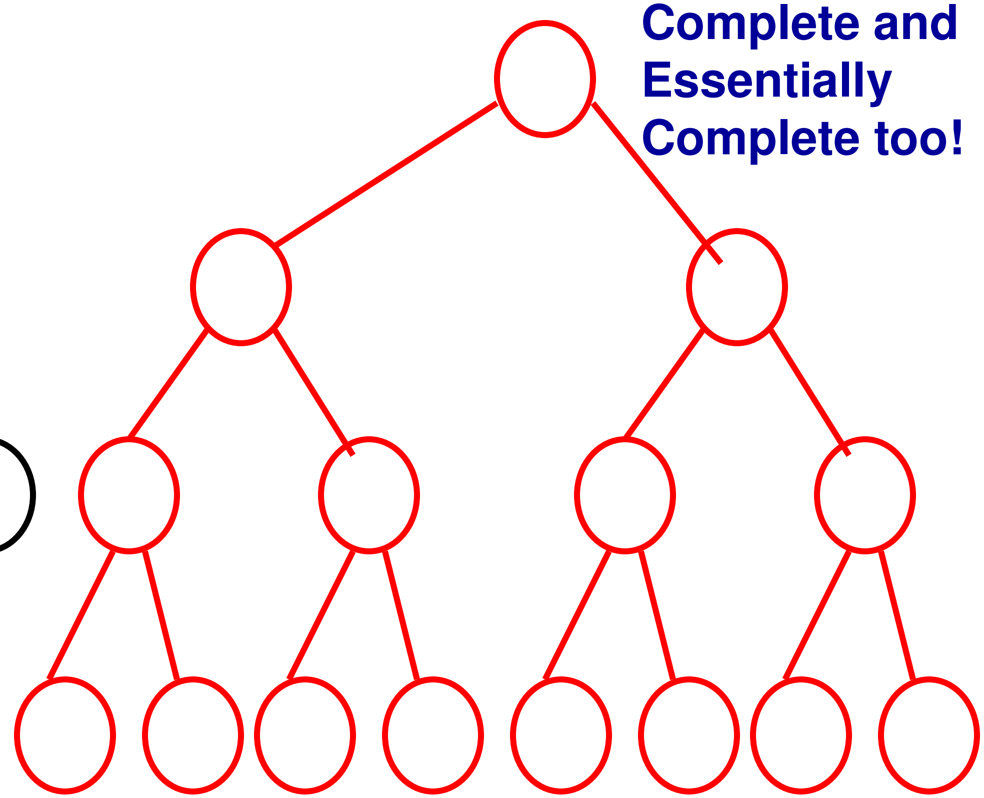
Essentially Complete Binary Tree

- A binary tree of height ‘h’ is essentially complete if it is a complete binary tree up to level h-1 and the nodes at level h are as far to the left as possible.
- Note: A complete binary tree is also essentially complete.

Essentially Complete

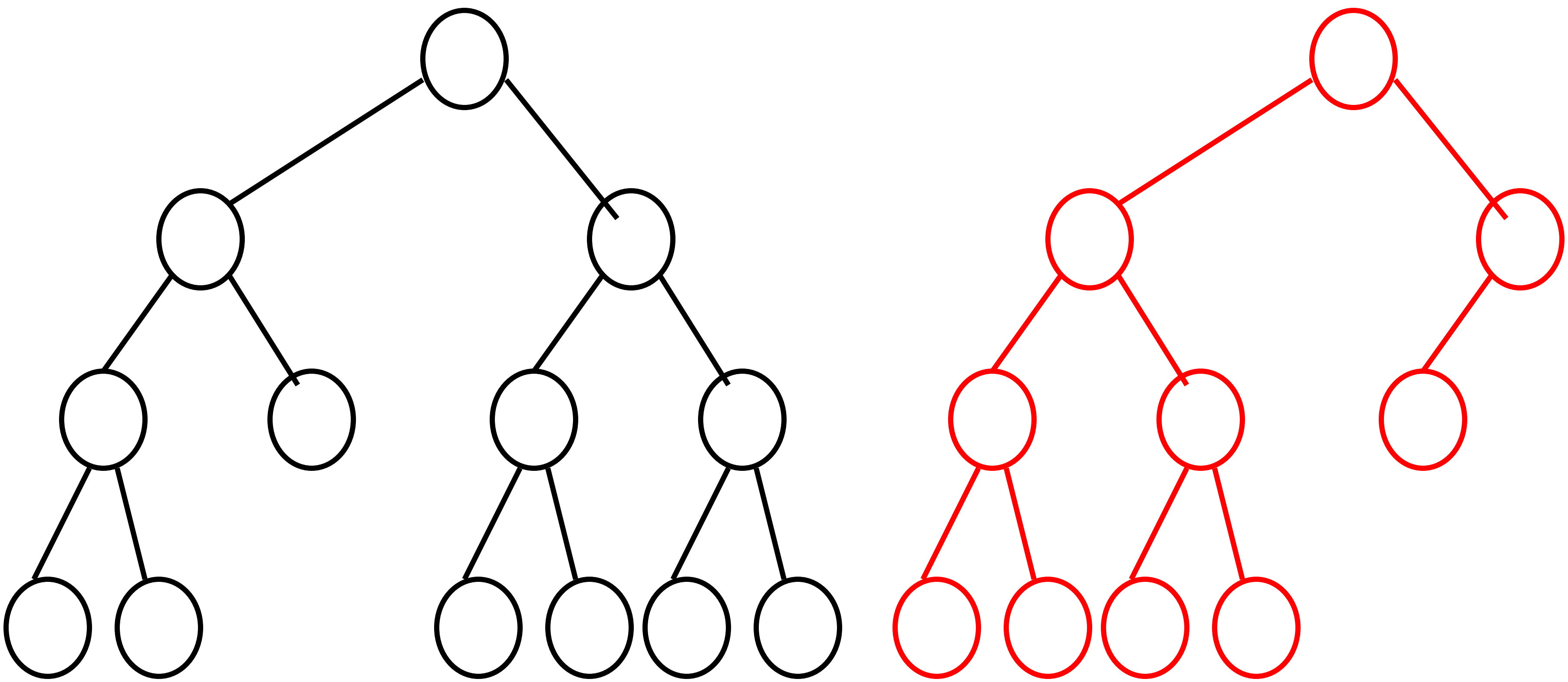


Complete and Essentially Complete too!



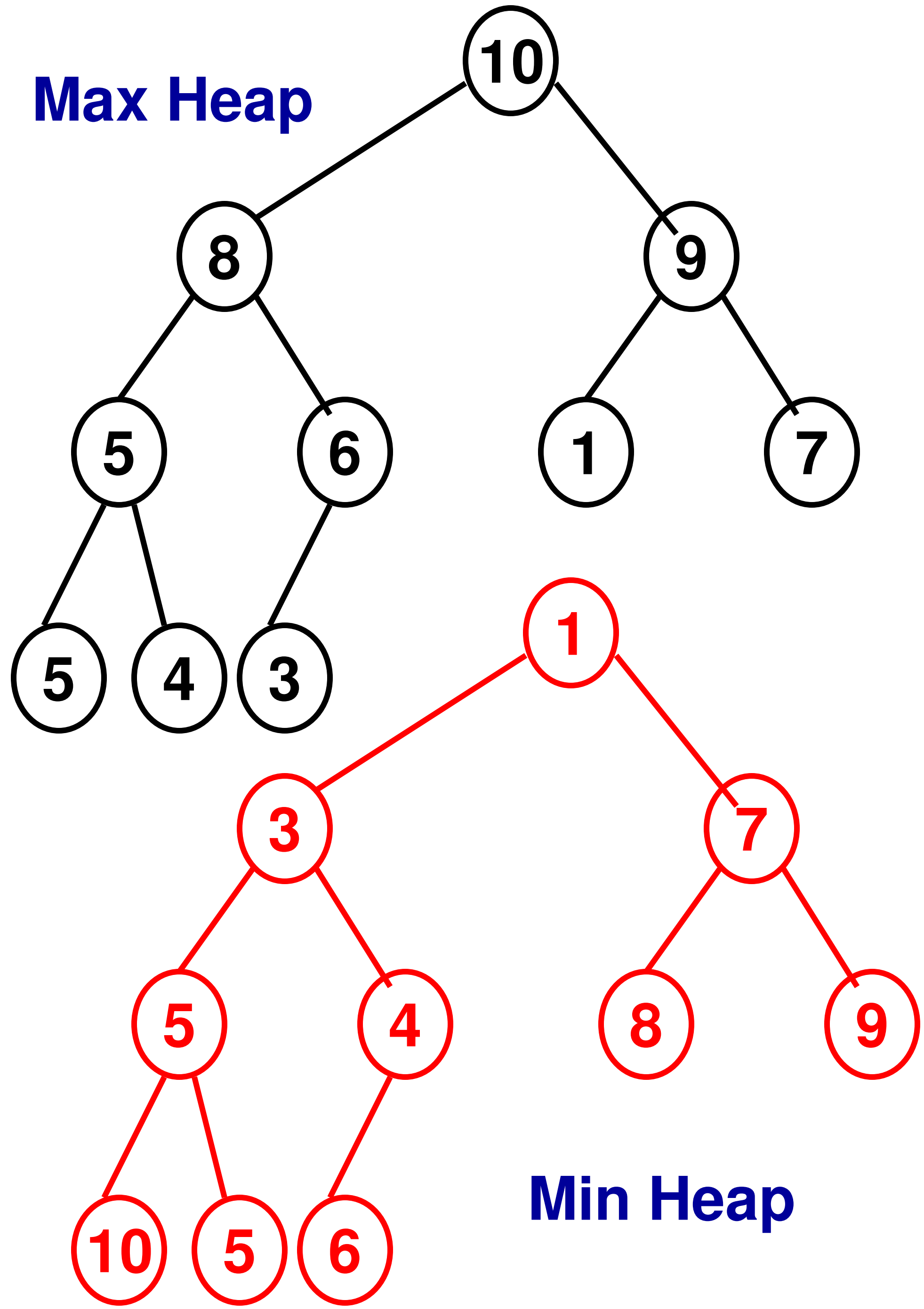
Essentially Complete Binary Tree

- The trees shown below are not essentially complete.



Heap

- A heap is a binary tree that satisfies the following two properties:
  - Essentially complete or complete
  - Max/Min heap
    - Max heap: The data at each internal node is greater than or equal to the data of its immediate child nodes
    - Min heap: The data at each internal node is lower than or equal to the data of its immediate child nodes

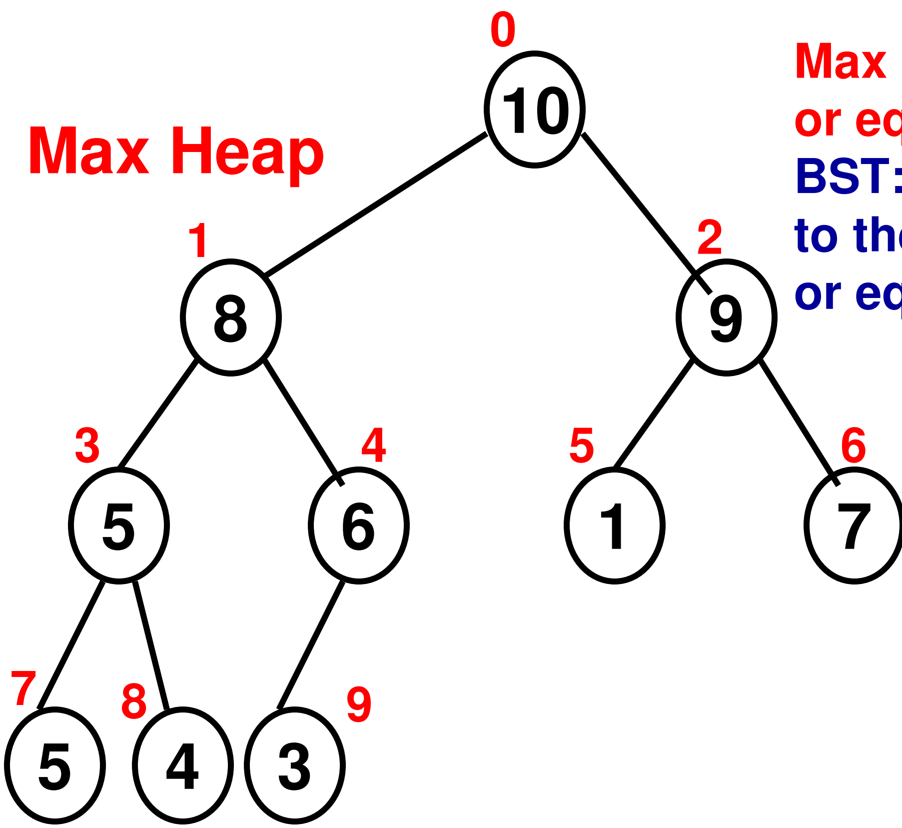


Difference between BST and Heap

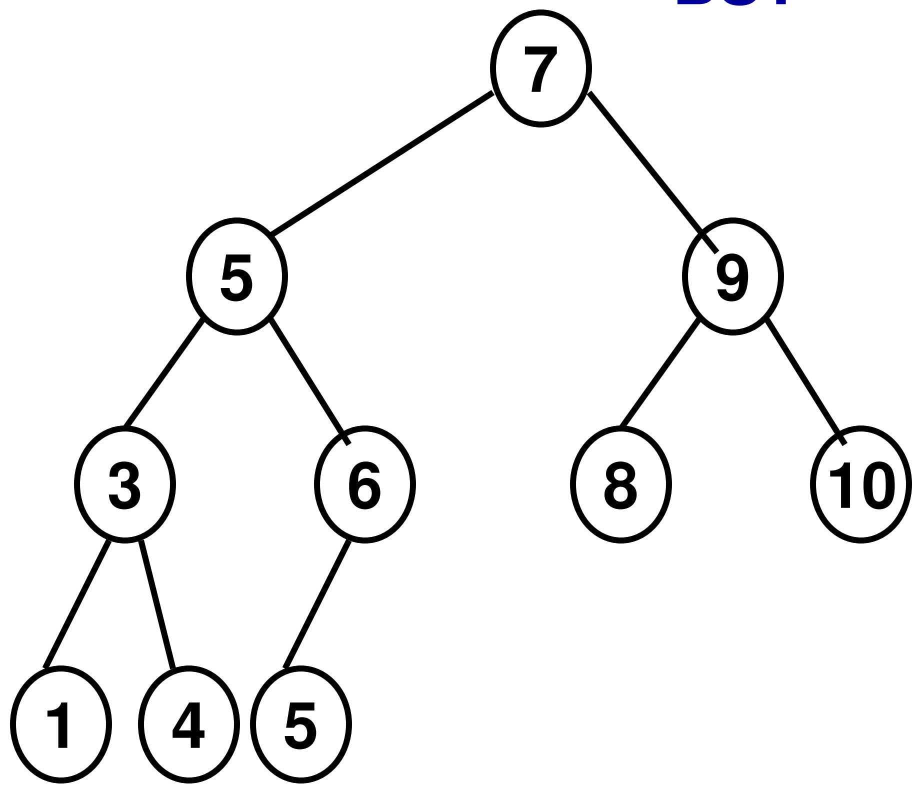
Max Heap: Each internal node is greater than or equal to its child nodes.

BST: An internal node is greater than or equal to the nodes in its left sub tree and lower than or equal to the nodes in its right sub tree.

Max Heap



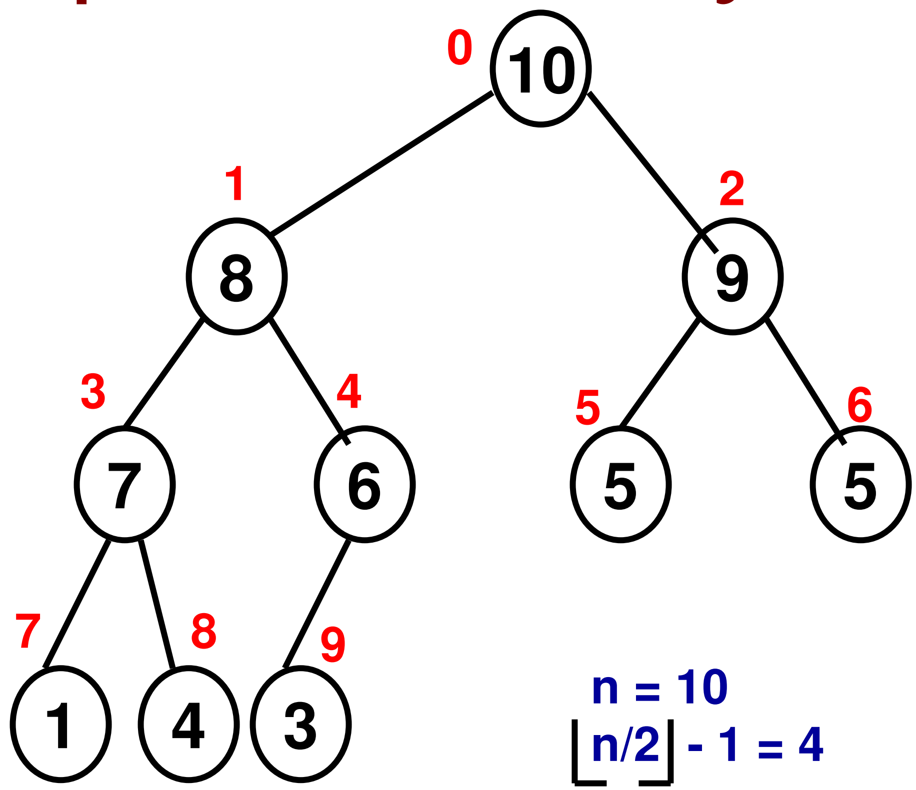
BST



7	**3**	**8**	**1**	**9**	**4**	**0**	**5**	**2**	**6**
5	5	4	8	3	6	10	1	9	7
1	3	4	5	5	6	7	8	9	10

Storing the Heap as an Array

- A heap of 'n' elements can be stored in an array (index starting from 0) such that the internal nodes (in the top-down, left-right order) are represented as elements from index 0 to $\lfloor n/2 \rfloor - 1$ and the leaf nodes (again, top-down, left-right order) are represented as elements from index $\lfloor n/2 \rfloor$ to n-1.
- The child nodes of an internal node at index 'j' are at indexes 2j+1 and 2j+2.
- The parent node for a node at index j is at index $\lfloor (j-1)/2 \rfloor$



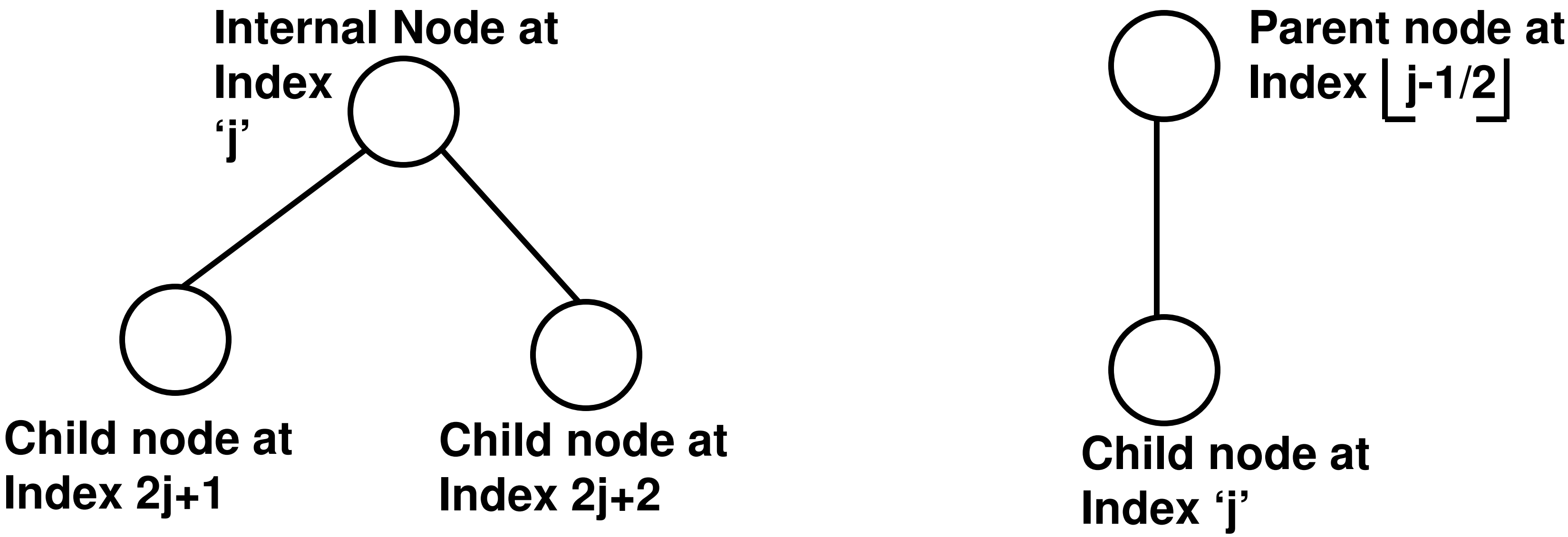
n = 10

$\lfloor n/2 \rfloor - 1 = 4$

0	1	2	3	4	5	6	7	8	9
10	8	9	7	6	5	5	1	4	3

The child nodes of internal node '8' at index 1 are at indexes 2*1+1 = 3 and 2*1 + 2 = 4. The parent node for node '7' at index 3 is at index $\lfloor (3-1)/2 \rfloor = 1$

Storing the Heap as an Array



For the rest of this module, we will construct and employ a 'max' heap unless otherwise specified.

> **The data for an internal node must be greater than or equal to that of its child nodes.**

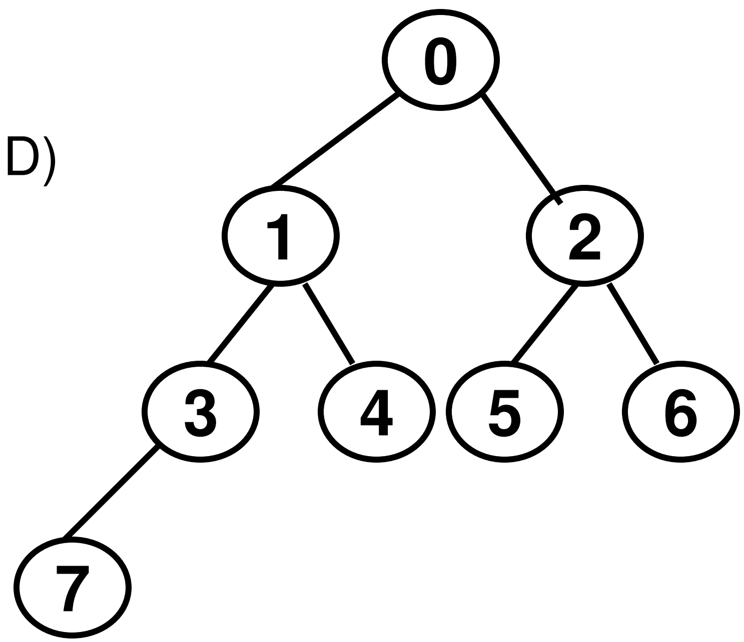
Using BFS to check whether a Binary Tree is Essentially Complete

We keep track of whether we come across a state wherein an internal node does not have a child node. The moment we come across an internal node with a missing Child node (left node or right node), we set the boolean 'noChildZoneStarts' to true If we across a child node when the noChildZoneStarts boolean is true, we declare the tree is not essentially complete!

```
Queue queue
queue.enqueue(root node id 0)
noChildZoneStarts = false

Begin BFS_BinaryTree
while (!queue.isEmpty()) do
  FirstNodeID = queue.dequeue();
  if (noChildZoneStarts == false AND FirstNode.leftChildNodeID == -1)
        noChildZoneStarts = true
  else if (noChildZoneStarts == true AND FirstNode.leftChildNodeID != -1)
        return "the binary tree is not essentially complete"

  if (FirstNode.leftChildNodeID != -1) then
     queue.enqueue(FirstNode.leftChildNodeID)
  end if
```

Breadth First Search (BFS) Algorithm continued…

```
    if (noChildZoneStarts == false AND FirstNode.rightChildNodeID == -1)
            noChildZoneStarts = true
    else if (noChildZoneStarts == true AND FirstNode.rightChildNodeID != -1)
            return “the binary tree is not essentially complete”

    if (FirstNode.rightChildNodeID != -1) then
      queue.enqueue(FirstNode.rightChildNodeID)
   end if

end while

return “the binary tree is essentially complete”

End BFS_BinaryTree
```



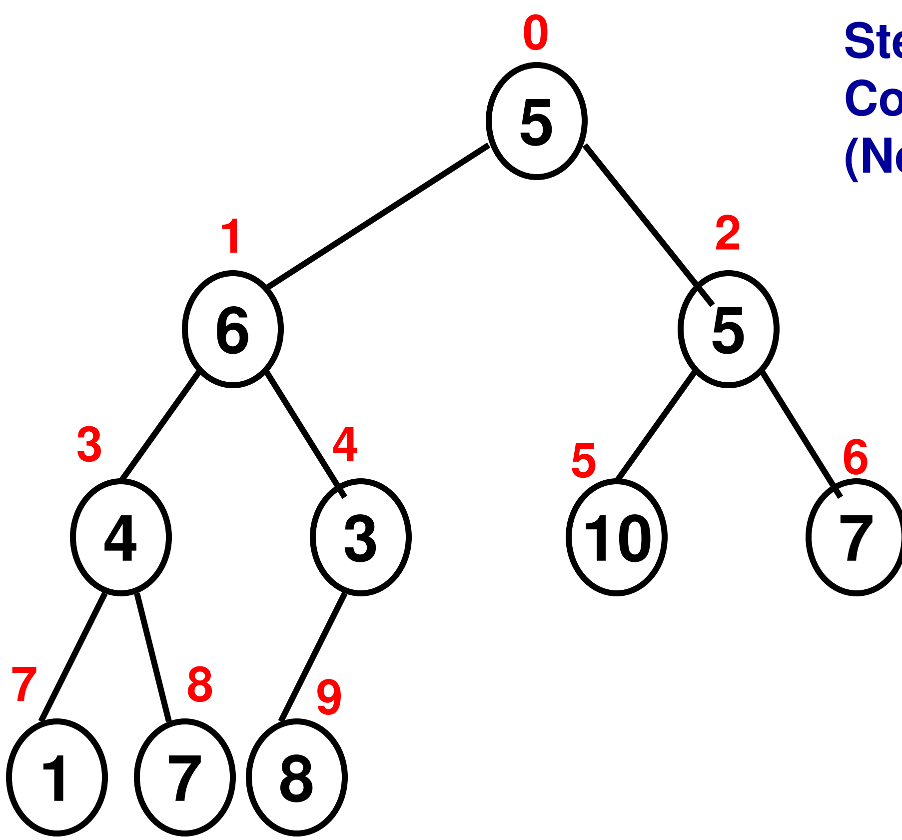
Once we find out that node ‘3’ does not have a right child, all the nodes explored further in BFS should not have any child node. Otherwise, the binary tree is not essentially complete.

Heap Construction

- Given an array of 'n' elements,
- Step 1: Construct an essentially complete binary tree and then reheapify the internal nodes of the tree to make sure the max or min heap property is satisfied for each internal node.
- Step 2: Reheapify an internal node for 'max' heap: If the data at an internal node is lower than that of one or both of its child nodes, then swap the data for the internal node with the larger of the data of its two child nodes.
  - If any internal node further down is affected because of this swap, the reheapify operation is recursively continued all the way until a leaf node is reached.
- The reheapify operation is started from the node at index $\lfloor n/2 \rfloor - 1$ and continued all the way to the node at index 0.

Heap Construction Example 1

0	1	2	3	4	5	6	7	8	9
5	6	5	4	3	10	7	1	7	8

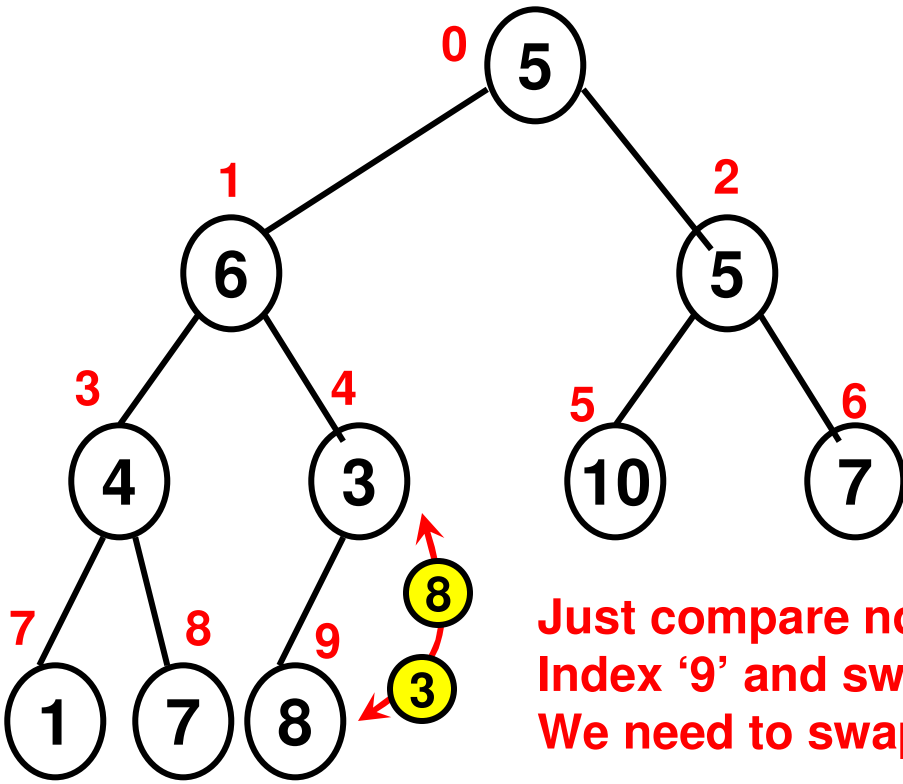


Step 1: Essentially Complete Binary Tree (Not a heap yet!)

Heap Construction Example 1

Before (Reheapify at Index '4'):

0	1	2	3	4	5	6	7	8	9
5	6	5	4	3	10	7	1	7	8



Step 2: Reheapify node at index '4' and down further if needed

Compare the node at index '4' with its child nodes at index 2*4 + 1 = 9 and index 2*4 + 2 = 10. Since index '10' does not exist and index 9 exists, it implies we have reached a leaf node (at index 9) and there is no need to proceed further down.

Just compare node at index '4' with the child node at Index '9' and swap them, if needed. In this case: Yes, We need to swap.

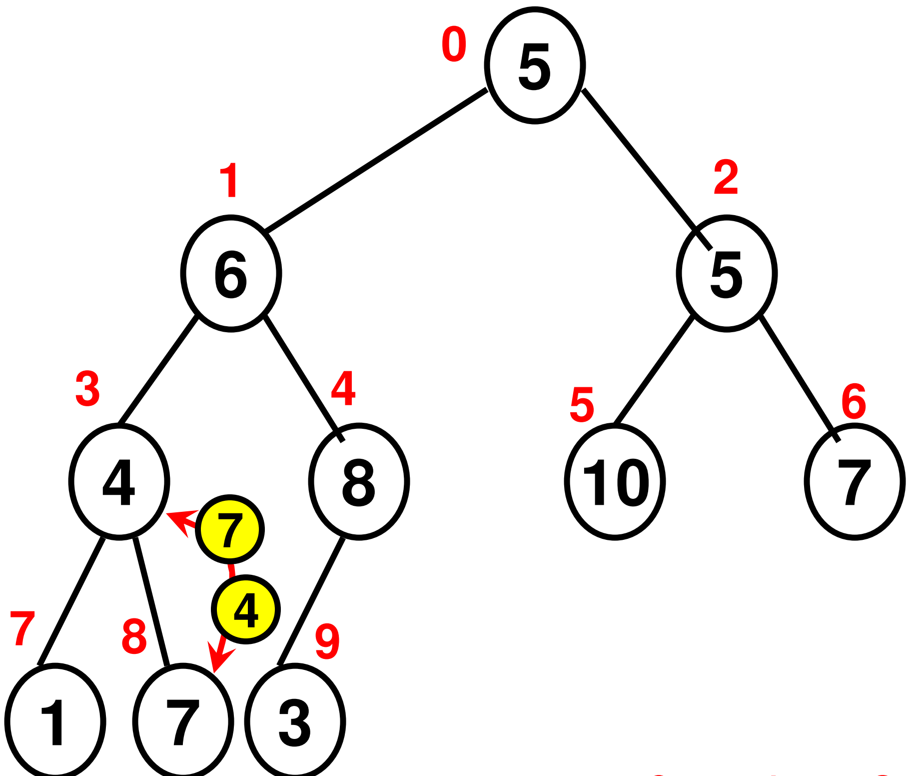
After (Reheapify at Index '4'):

0	1	2	3	4	5	6	7	8	9
5	6	5	4	8	10	7	1	7	3

Heap Construction Example 1

Before (Reheapify at Index ‘3’):

0	1	2	3	4	5	6	7	8	9
5	6	5	4	8	10	7	1	7	3



Step 2: Reheapify node at index ‘3’ and down further if needed

Compare the node at index ‘3’ with its child nodes at index 2*3 + 1 = 7 and index 2*3 + 2 = 8. In this case, We swap element at index ‘3’ with element at index ‘8’. Since 8 is already a leaf node, we do not proceed down further.

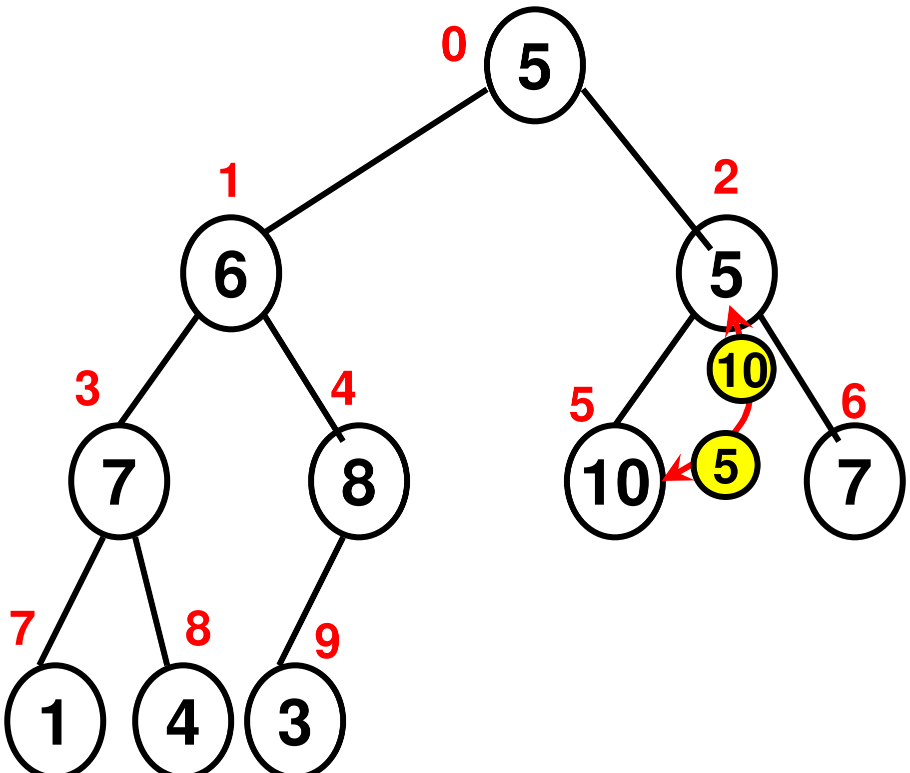
After (Reheapify at Index ‘3’):

0	1	2	3	4	5	6	7	8	9
5	6	5	7	8	10	7	1	4	3

Heap Construction Example 1

Before (Reheapify at Index '2'):

0	1	2	3	4	5	6	7	8	9
5	6	5	7	8	10	7	1	4	3



Step 2: Reheapify node at index '2' and down further if needed

Compare the node at index '2' with its child nodes at index 2*2 + 1 = 5 and index 2*2 + 2 = 6. In this case, We swap element at index '2' with element at index '5'. Since 5 is already a leaf node, we do not proceed down further.

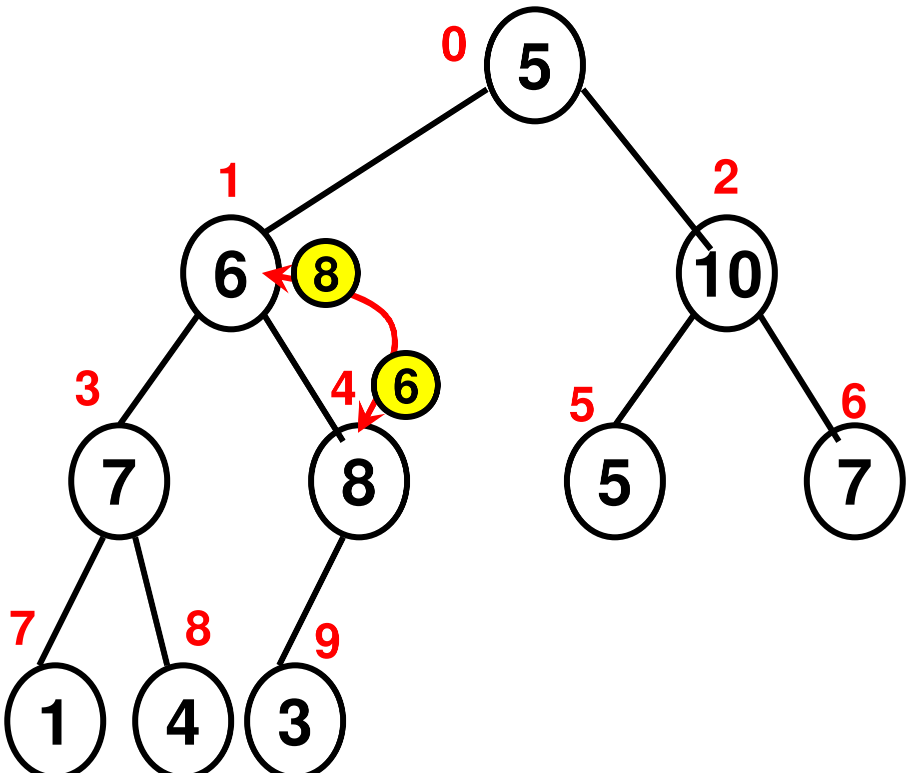
After (Reheapify at Index '2'):

0	1	2	3	4	5	6	7	8	9
5	6	10	7	8	5	7	1	4	3

Heap Construction Example 1

Before (Reheapify at Index ‘1’):

0	1	2	3	4	5	6	7	8	9
5	6	10	7	8	5	7	1	4	3



Step 2: Reheapify node at index ‘1’ and down further if needed

Compare the node at index ‘1’ with its child nodes at index 2*1 + 1 = 3 and index 2*1 + 2 = 4. In this case, We swap element at index ‘1’ with element at index ‘4’.

Again do a reheapify at index ‘4’, if needed and continue in a recursive fashion until it is no longer needed.

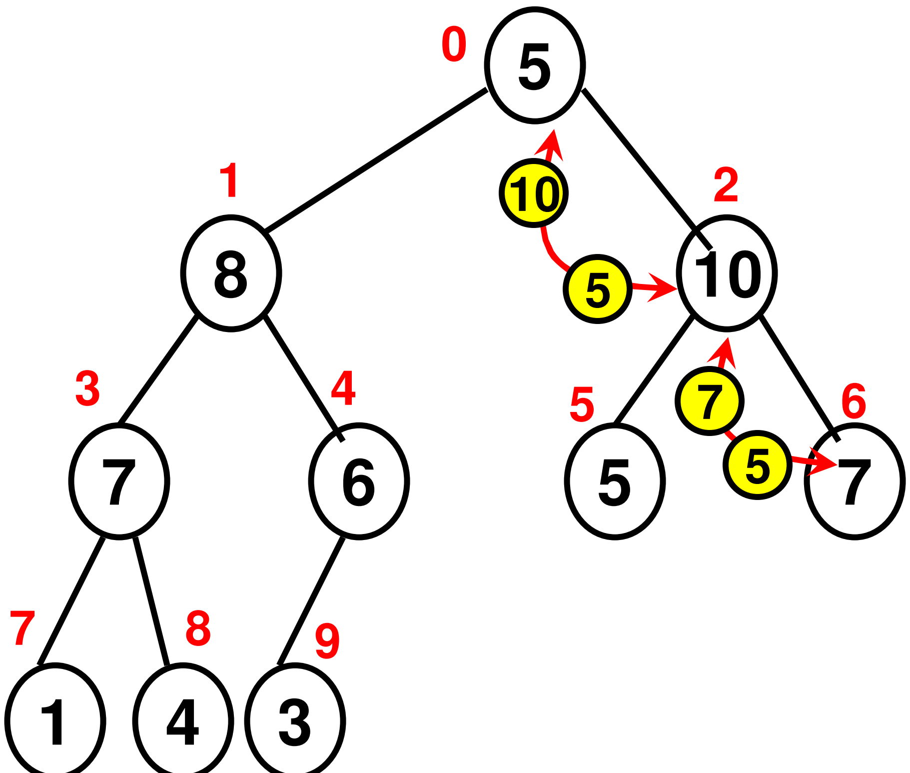
After (Reheapify at Index ‘1’):

0	1	2	3	4	5	6	7	8	9
5	8	10	7	6	5	7	1	4	3

Heap Construction Example 1

Before (Reheapify at Index ‘0’):

0	1	2	3	4	5	6	7	8	9
5	8	10	7	6	5	7	1	4	3



Step 2: Reheapify node at index ‘0’ and down further if needed

Compare the node at index ‘0’ with its child nodes at index 2*0 + 1 = 1 and index 2*0 + 2 = 2. In this case, We swap element at index ‘0’ with element at index ‘2’.
Again do a reheapify at index ‘2’ as the element now at index ‘2’ (which is 5) is lower than the maximum of its two child nodes (which is 9 at index ‘6’).

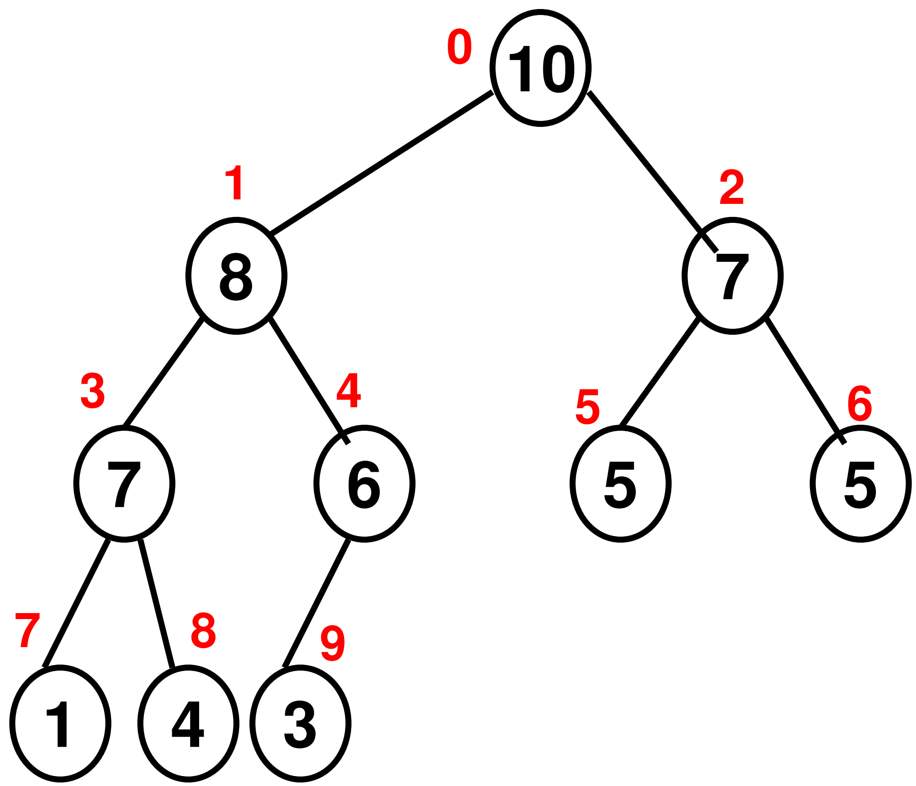
After (Reheapify at Index ‘0’):

0	1	2	3	4	5	6	7	8	9
10	8	7	7	6	5	5	1	4	3

Heap Construction Example 1

Final Array Representing Max Heap

0	1	2	3	4	5	6	7	8	9
10	8	7	7	6	5	5	1	4	3



Max Heap Construction (Code 8.1: C++)

Main Function

```
int arraySize;
cout << "Enter array size: ";
cin >> arraySize;

int array[arraySize];

int maxValue;
cout << "Enter the max. value for any element: ";
cin >> maxValue;

srand(time(NULL));

cout << "Generated array: ";
for (int i = 0; i < arraySize; i++){
   array[i] = rand() % maxValue;
   cout << array[i] << " ";
}
```

```
//max. heap construction

for (int index = (arraySize/2)-1; index  >= 0; index--)
        rearrangeHeapArray(array, arraySize, index);
}


cout << "After Heap construction..." << endl;
for (int index = 0; index < arraySize; index++)
        cout  << array[index] << " ";

cout << endl;
```

7.1: Reheapify Code (C++)

```
void rearrangeHeapArray(int *array, int arraySize, int index){
        // max heap construction

        int leftChildIndex = 2*index + 1;
        int rightChildIndex = 2*index + 2;
                // If the node at 'index' does not have a left child (implies it does
        if (leftChildIndex >= arraySize)    // not have right child too), then there
                return;                     // is no need to reheapify at that index

        // If the node at 'index' does not have a right child (if the control reaches
        if (rightChildIndex >= arraySize){      // here, it implies the node
// Check if the data for the                    // at 'index' has a left child)
// node at      if (array[index] < array[leftChildIndex]){
// 'index' is less      int temp = array[index];
// than that of its     array[index] = array[leftChildIndex];
// left child. If so,   array[leftChildIndex] = temp;
// swap         }

                return;
        }
```

// If the control reaches here, it means the node at ‘index’ has both left child
// and right child

// If the node at ‘index’ has data that is greater than or equal to both its left child and right child, then there is no need to reheapify for this index

```
if (array[index] >= array[leftChildIndex] &&
    array[index] >=  array[rightChildIndex])
        return;
```

// If the control reaches here, it implies the node at ‘index’ has data that is less than at least one of its two child nodes

```
int maxIndex = leftChildIndex;
if (array[leftChildIndex] < array[rightChildIndex])
        maxIndex = rightChildIndex;
```

// Between the left and right
// child nodes, find the node
// that has relatively larger
// data, call the index of this
// as ‘maxIndex’ and swap
// its value with the node at
// ‘index’.

```
int temp = array[maxIndex];
array[maxIndex] = array[index];
array[index] = temp;

rearrangeHeapArray(array, arraySize, maxIndex);
```

// Call the rearrangeHeap function in a recursive fashion
// to see if further rearrangements need to be done starting
// from maxIndex

Max Heap Construction (Code 8.1: Java)

Main Function

```
Scanner input = new Scanner(System.in);

int arraySize;
System.out.print("Enter array size: ");
arraySize = input.nextInt();

int array[] = new int[arraySize];

int maxValue;
System.out.print("Enter the max. value for any element: ");
maxValue = input.nextInt();

Random randGen = new Random(System.currentTimeMillis());

System.out.print("Generated array: ");
for (int i = 0; i < arraySize; i++){
        //System.out.print("Enter element at index " + i + " : ");
        //array[i] = input.nextInt();
        array[i] = randGen.nextInt(maxValue);
        System.out.print(array[i] + " ");
}

System.out.println();
```

Max Heap Construction (Code 8.1: Java)

Main Function

```java
//max. heap construction

for (int index = (arraySize/2)-1; index >= 0; index--){
        rearrangeHeapArray(array, arraySize, index);
 }

System.out.println("After Heap construction...");
for (int index = 0; index < arraySize; index++)
        System.out.print(array[index] + " ");

System.out.println();
```

7.1: Reheapify Code (Java)

```
public static void rearrangeHeapArray(int array[], int arraySize, int index){
        // max heap construction

        int leftChildIndex = 2*index + 1;
        int rightChildIndex = 2*index + 2;
                // If the node at 'index' does not have a left child (implies it does
        if (leftChildIndex >= arraySize)   // not have right child too), then there
                return;                    // is no need to reheapify at that index

        // If the node at 'index' does not have a right child (if the control reaches
        if (rightChildIndex >= arraySize){       // here, it implies the node
                                                 // at 'index' has a left child)
// Check if the data for the
// node at      if (array[index] < array[leftChildIndex]){
// 'index' is less      int temp = array[index];
// than that of its     array[index] = array[leftChildIndex];
// left child. If so,   array[leftChildIndex] = temp;
// swap         }

                return;
        }
```

// If the control reaches here, it means the node at ‘index’ has both left child
// and right child

```
if (array[index] >= array[leftChildIndex] && array[index] >=
                        array[rightChildIndex])
        return;
```

// If the node at ‘index’ has data that is greater than or equal to both its left child and right child, then there is no need to reheapify for this index

// If the control reaches here, it implies the node at ‘index’ has data that is less than at least one of its two child nodes

```
int maxIndex = leftChildIndex;
if (array[leftChildIndex] < array[rightChildIndex])
        maxIndex = rightChildIndex;

int temp = array[maxIndex];
array[maxIndex] = array[index];
array[index] = temp;
```

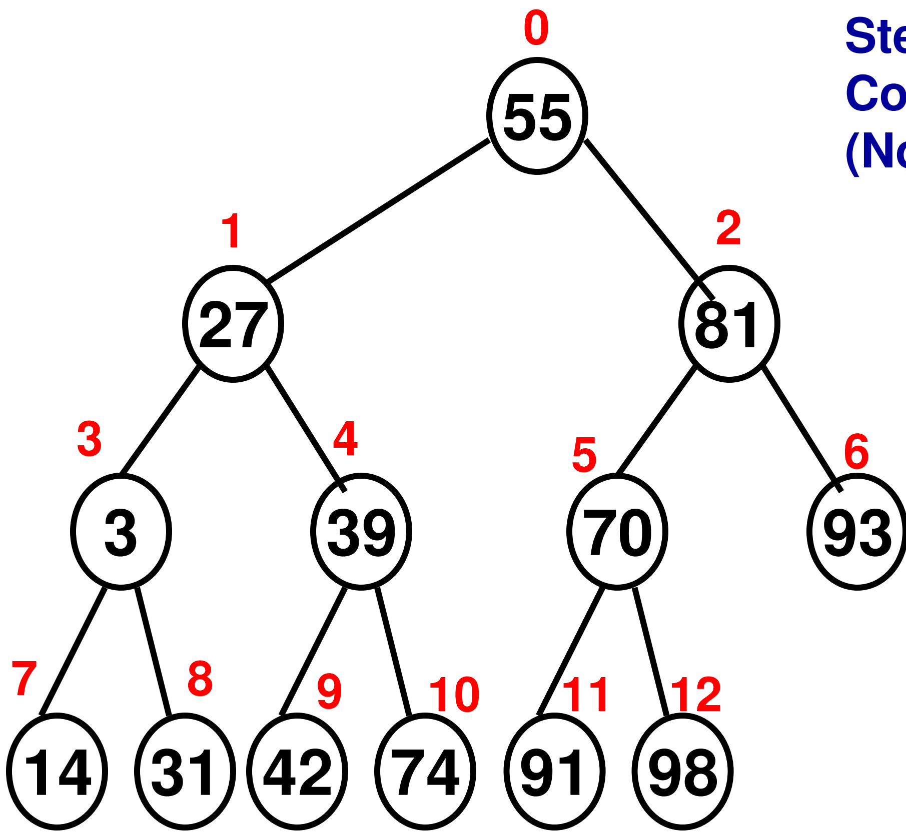
// Between the left and right
// child nodes, find the node
// that has relatively larger
// data, call the index of this
// as ‘maxIndex’ and swap
// its value with the node at
// ‘index’.

```
rearrangeHeapArray(array, arraySize, maxIndex);
```

// Call the rearrangeHeap function in a recursive fashion
// to see if further rearrangements need to be done starting
// from maxIndex

Heap Construction Example 2

0	1	2	3	4	5	6	7	8	9	10	11	12
55	27	81	3	39	70	93	14	31	42	74	91	98



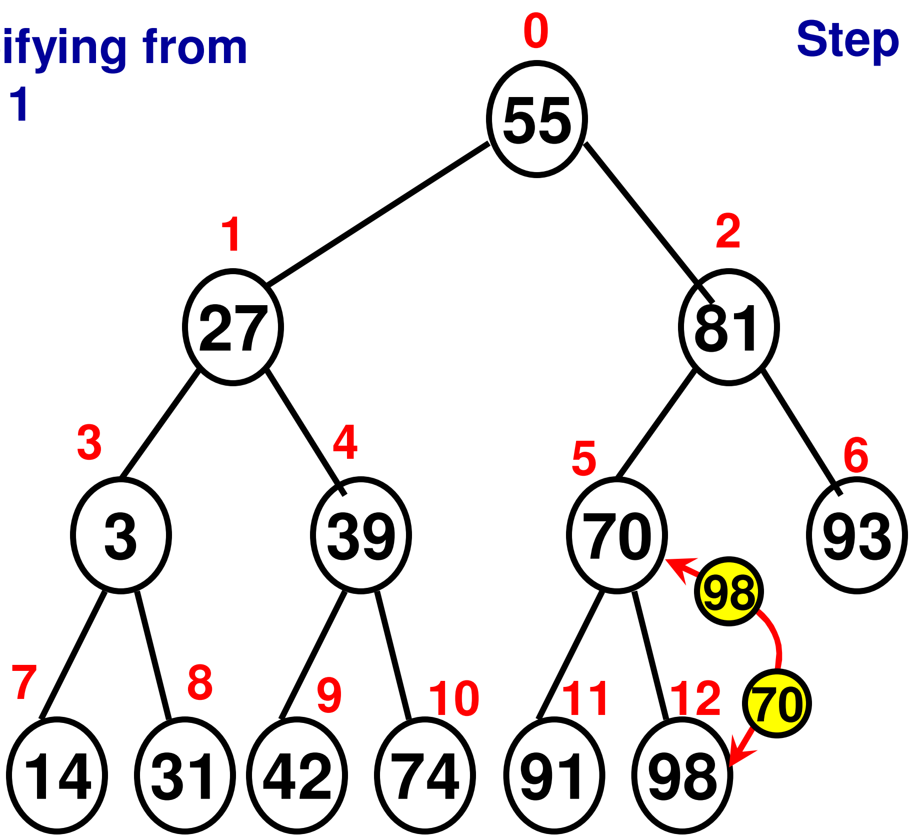
Step 1: Essentially Complete Binary Tree (Not a heap yet!)

Before (Reheapify at Index ‘5’):

0	1	2	3	4	5	6	7	8	9	10	11	12
55	27	81	3	39	70	93	14	31	42	74	91	98

Start reheapifying from Index $\lfloor 13/2 \rfloor - 1$

Step 2: Reheapify at index 5



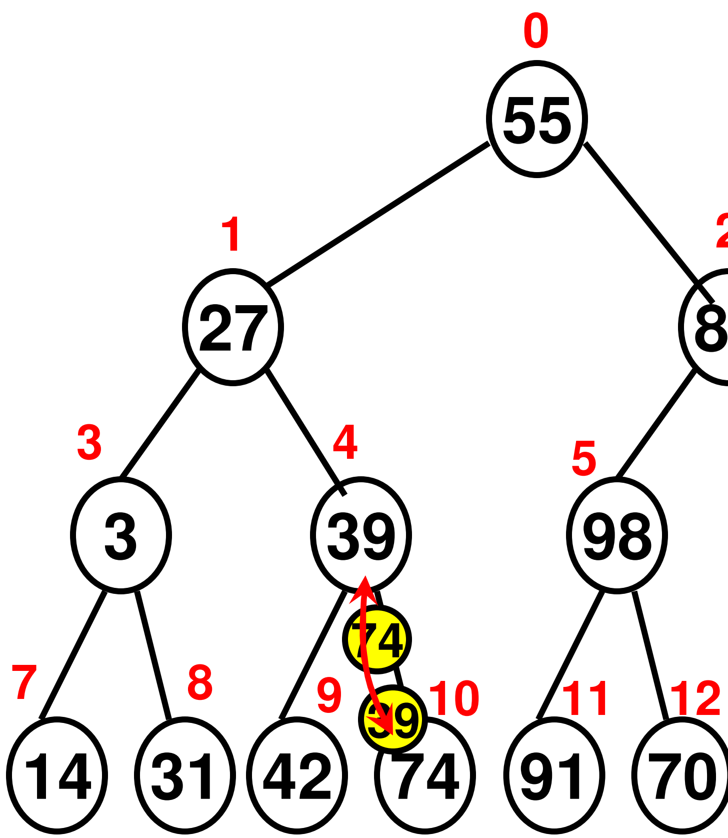
After (Reheapify at Index ‘5’):

0	1	2	3	4	5	6	7	8	9	10	11	12
55	27	81	3	39	98	93	14	31	42	74	91	70

Before (Reheapify at Index '4'):

0	1	2	3	4	5	6	7	8	9	10	11	12
55	27	81	3	39	98	93	14	31	42	74	91	70

Step 2: Reheapify at index 4



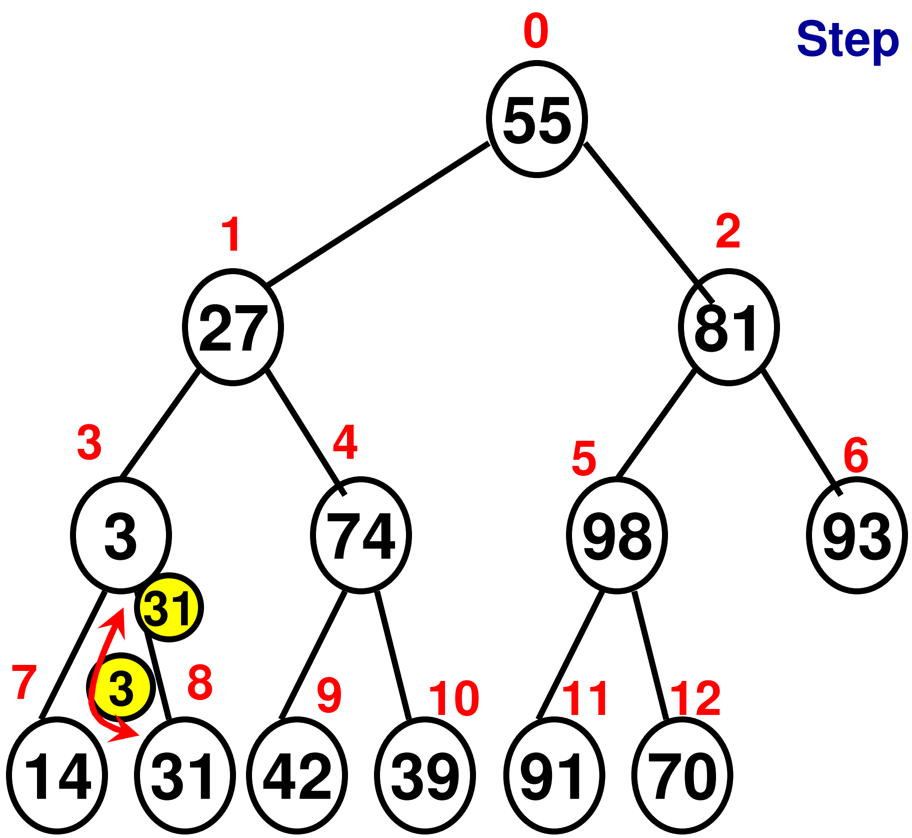
After (Reheapify at Index '4'):

0	1	2	3	4	5	6	7	8	9	10	11	12
55	27	81	3	74	98	93	14	31	42	39	91	70

Before (Reheapify at Index ‘3’):

0	1	2	3	4	5	6	7	8	9	10	11	12
55	27	81	3	74	98	93	14	31	42	39	91	70

Step 2: Reheapify at index 3



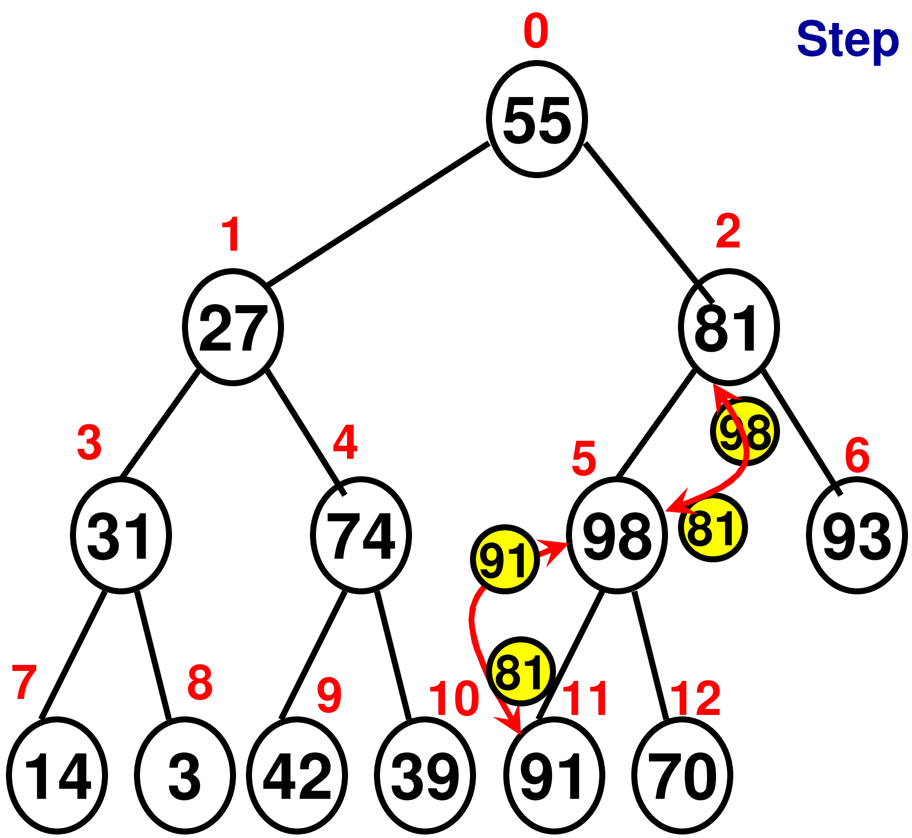
After (Reheapify at Index ‘3’):

0	1	2	3	4	5	6	7	8	9	10	11	12
55	27	81	31	74	98	93	14	3	42	39	91	70

Before (Reheapify at Index ‘2’):

0	1	2	3	4	5	6	7	8	9	10	11	12
55	27	81	31	74	98	93	14	3	42	39	91	70

Step 2: Reheapify at index 2



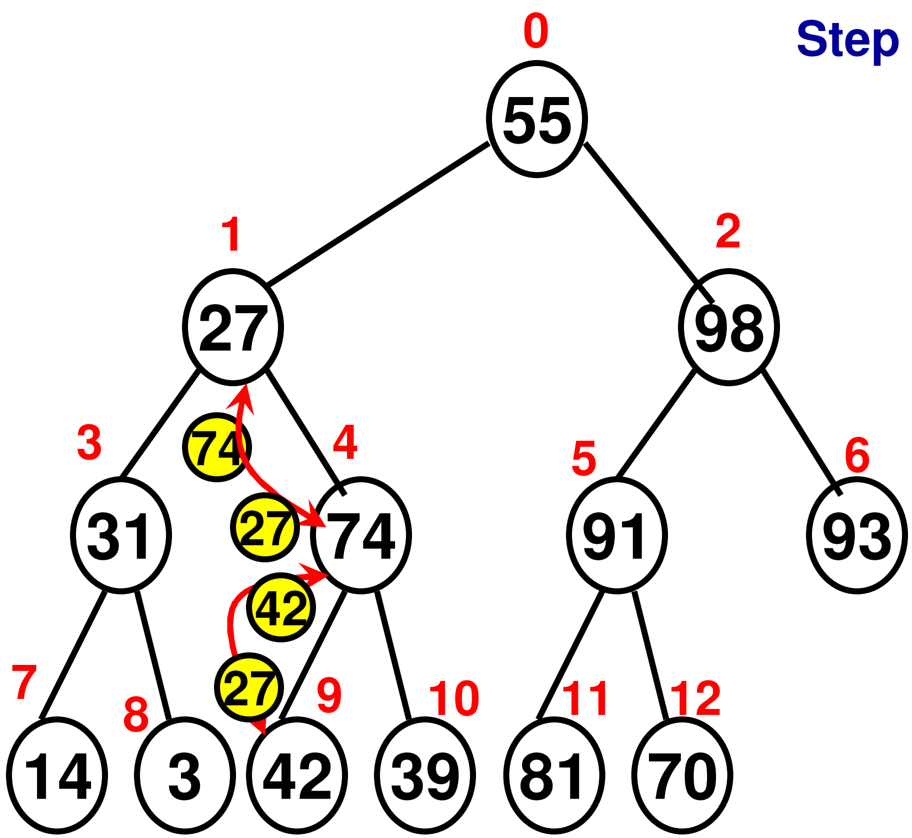
After (Reheapify at Index ‘2’):

0	1	2	3	4	5	6	7	8	9	10	11	12
55	27	98	31	74	91	93	14	3	42	39	81	70

Before (Reheapify at Index ‘1’):

0	1	2	3	4	5	6	7	8	9	10	11	12
55	27	98	31	74	91	93	14	3	42	39	81	70

Step 2: Reheapify at index 1



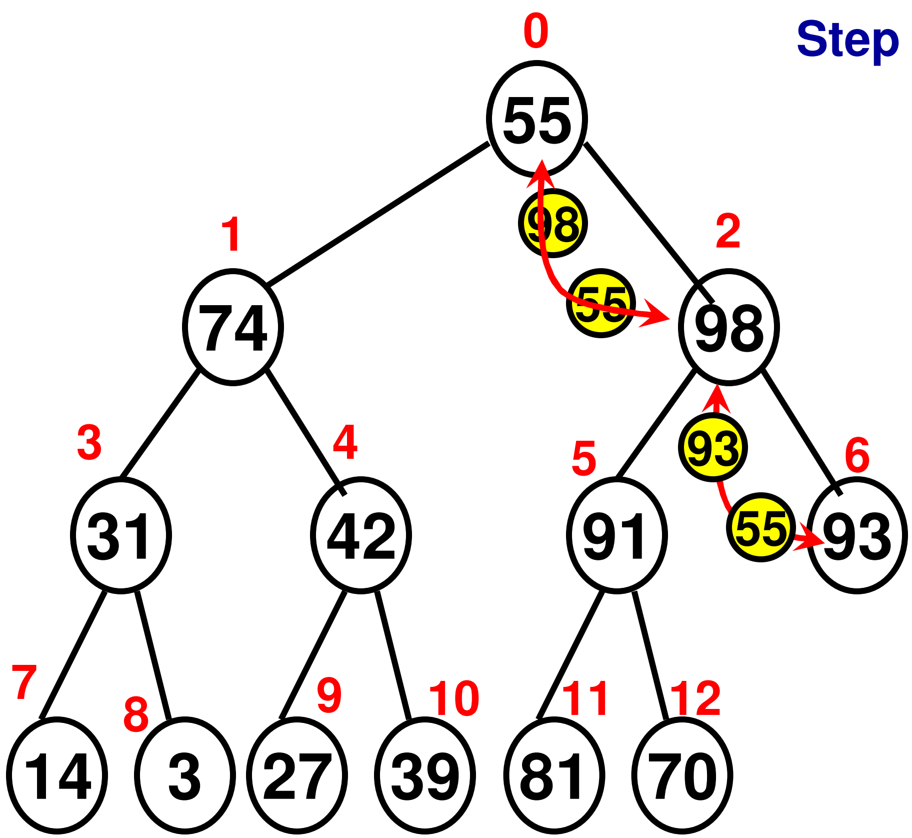
After (Reheapify at Index ‘1’):

0	1	2	3	4	5	6	7	8	9	10	11	12
55	74	98	31	42	91	93	14	3	27	39	81	70

Before (Reheapify at Index ‘0’):

0	1	2	3	4	5	6	7	8	9	10	11	12
55	74	98	31	42	91	93	14	3	27	39	81	70

Step 2: Reheapify at index 0



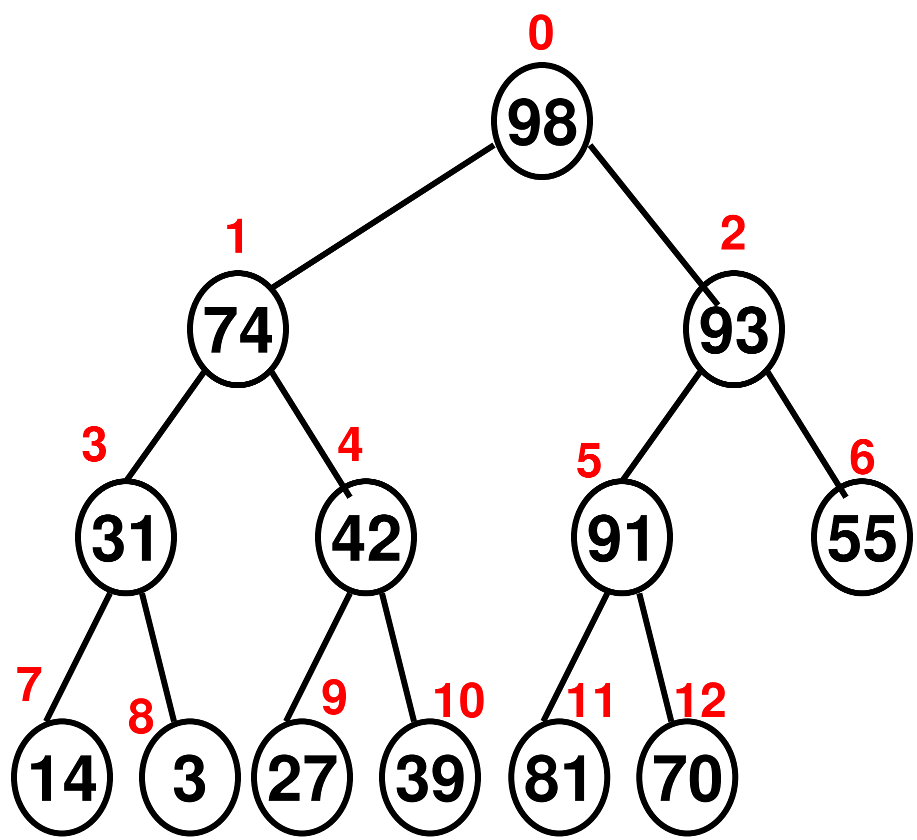
After (Reheapify at Index ‘0’):

0	1	2	3	4	5	6	7	8	9	10	11	12
98	74	93	31	42	91	55	14	3	27	39	81	70

Heap Construction Example 2

0	1	2	3	4	5	6	7	8	9	10	11	12
98	74	93	31	42	91	55	14	3	27	39	81	70

Final Array Representing Max Heap



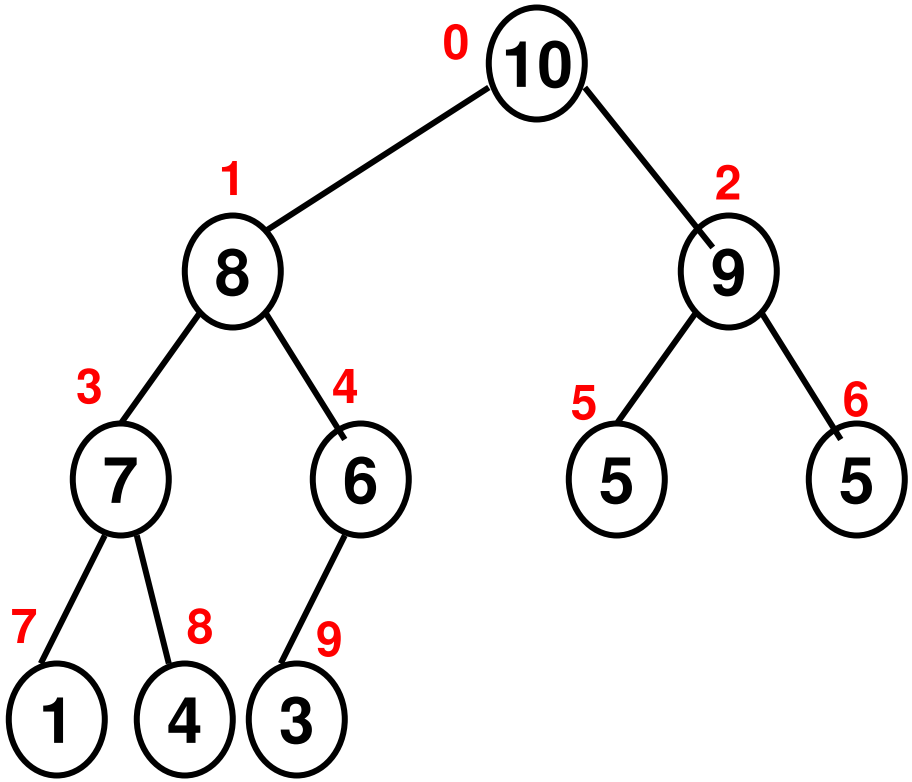
Heap Sort

- Given an array of size ‘n’, first construct a max-heap version of the array.
- Run ‘n-1’ iterations (iteration index 0 to n-1)
  - Swap element at index “0” with element at index “n-1-iteration index”
  - Element at index “0” has now moved to its final location “n-1-iteration index” in the sorted array
  - Reheapify the array as a result of this swap with the array index values ranging from “0” to “n-1-iteration index – 1”.
- Each iteration would require “logn” swappings at the worst case, across the entire height of the binary tree.
- For a total of ‘n-1’ iterations, the time complexity of heap sort is O(nlogn).

Heap Sort: Example 1

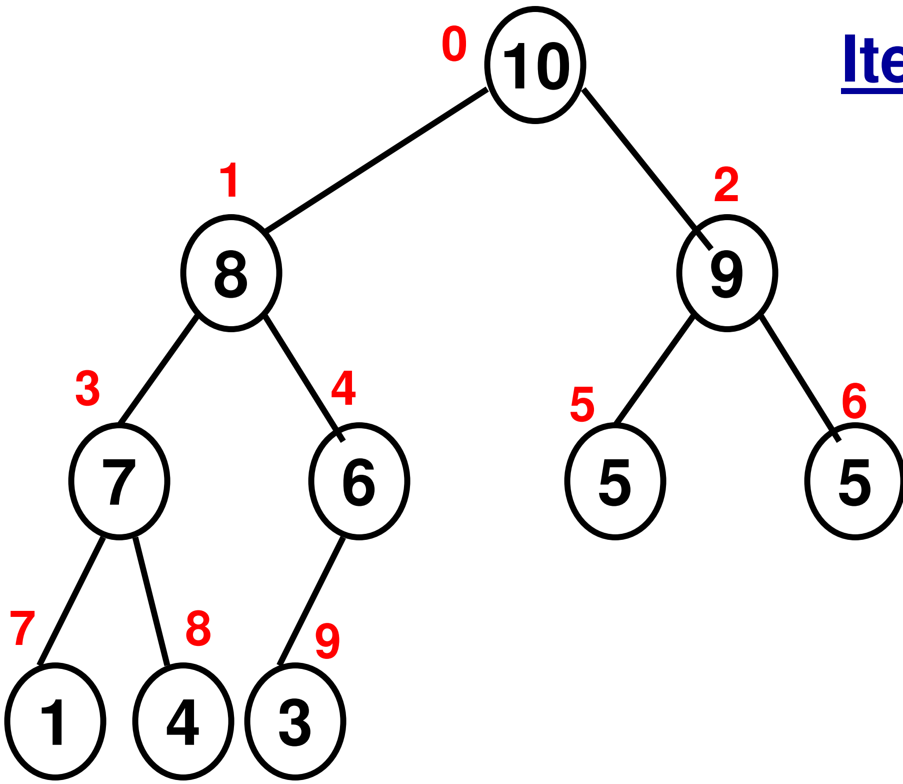
Original Array	5	6	5	4	3	10	9	1	7	8
	0	1	2	3	4	5	6	7	8	9
Max-Heap Version	10	8	9	7	6	5	5	1	4	3

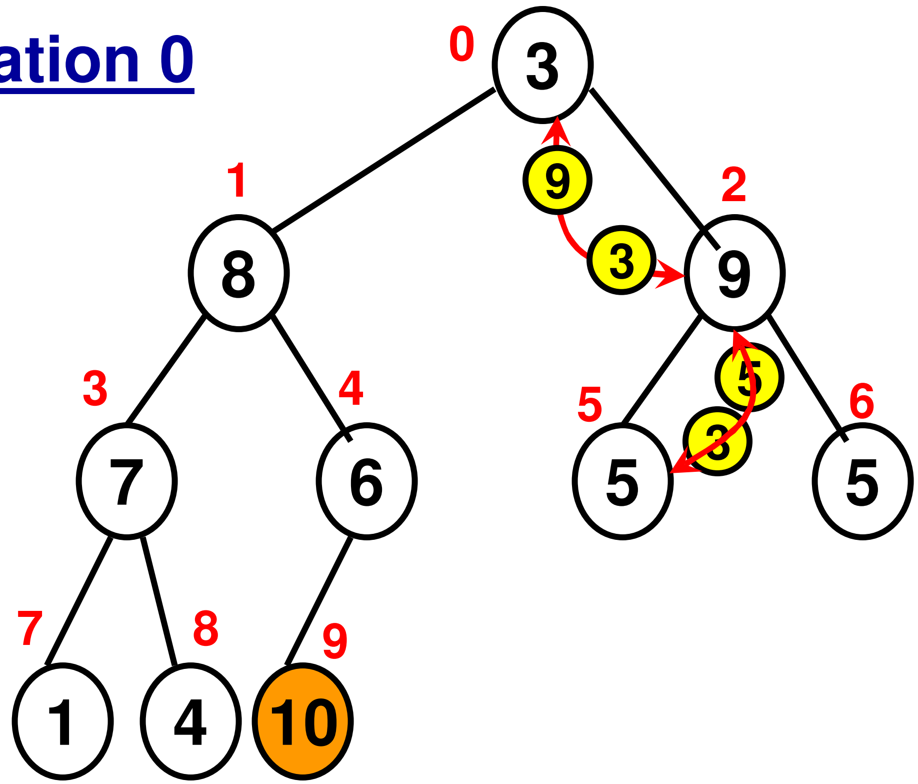
Max-Heap



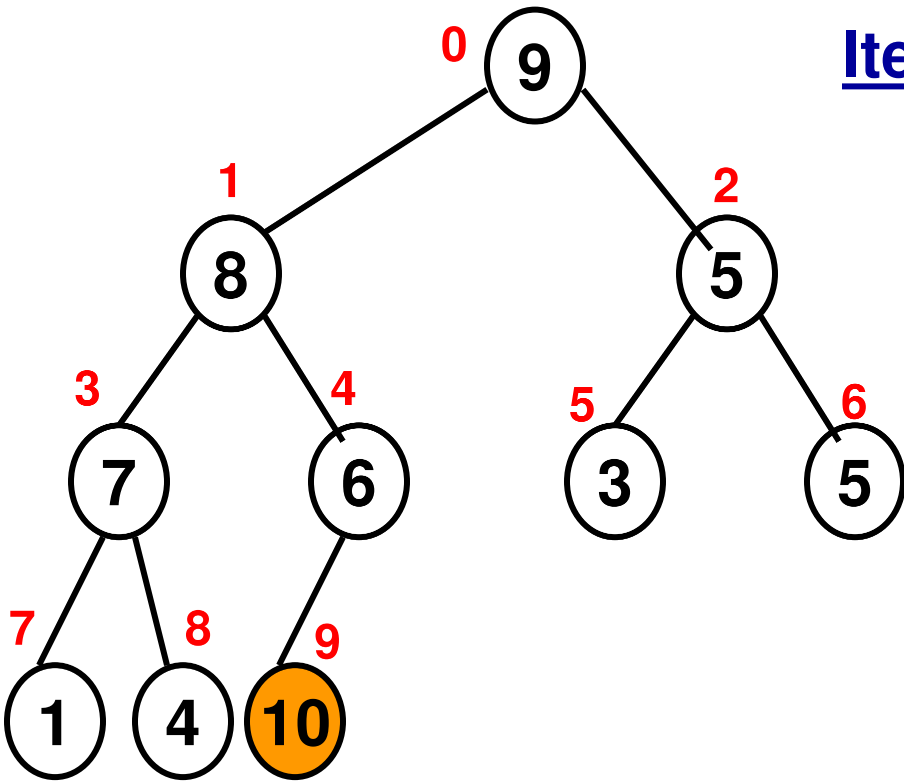
	0	1	2	3	4	5	6	7	8	9
Max-Heap Version	10	8	9	7	6	5	5	1	4	3
After Swap	3	8	9	7	6	5	5	1	4	10
After Reheapify	9	8	5	7	6	3	5	1	4	10

Iteration 0

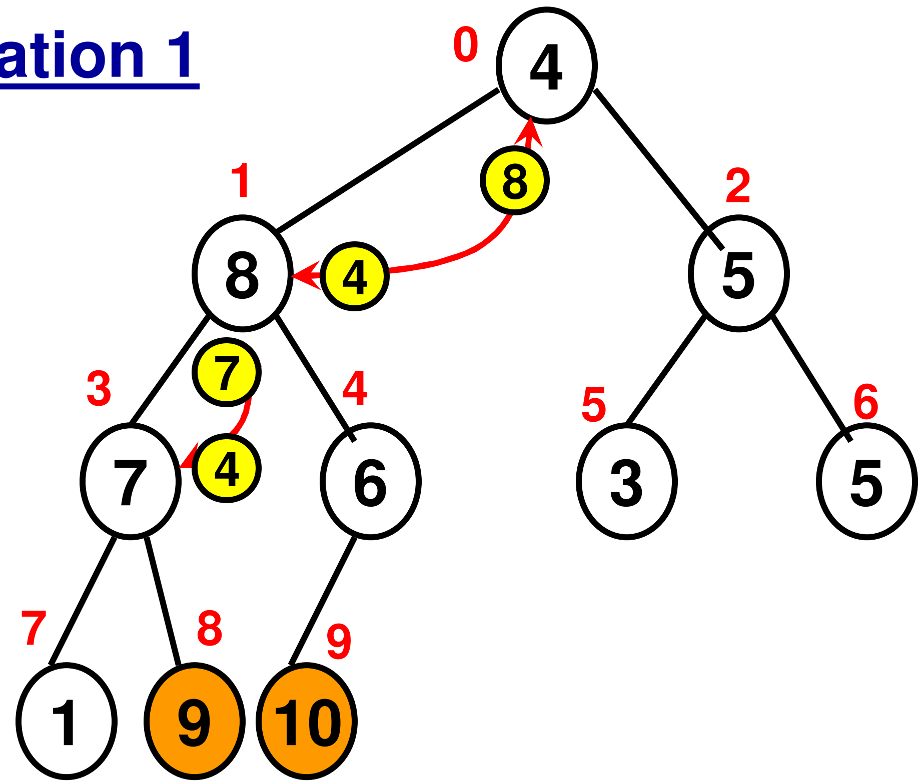




	0	1	2	3	4	5	6	7	8	9
Max-Heap Version	9	8	5	7	6	3	5	1	4	10
After Swap	4	8	5	7	6	3	5	1	9	10
After Reheapify	8	7	5	4	6	3	5	1	9	10



Iteration 1



Max-Heap Version

0	1	2	3	4	5	6	7	8	9
8	7	5	4	6	3	5	1	9	10

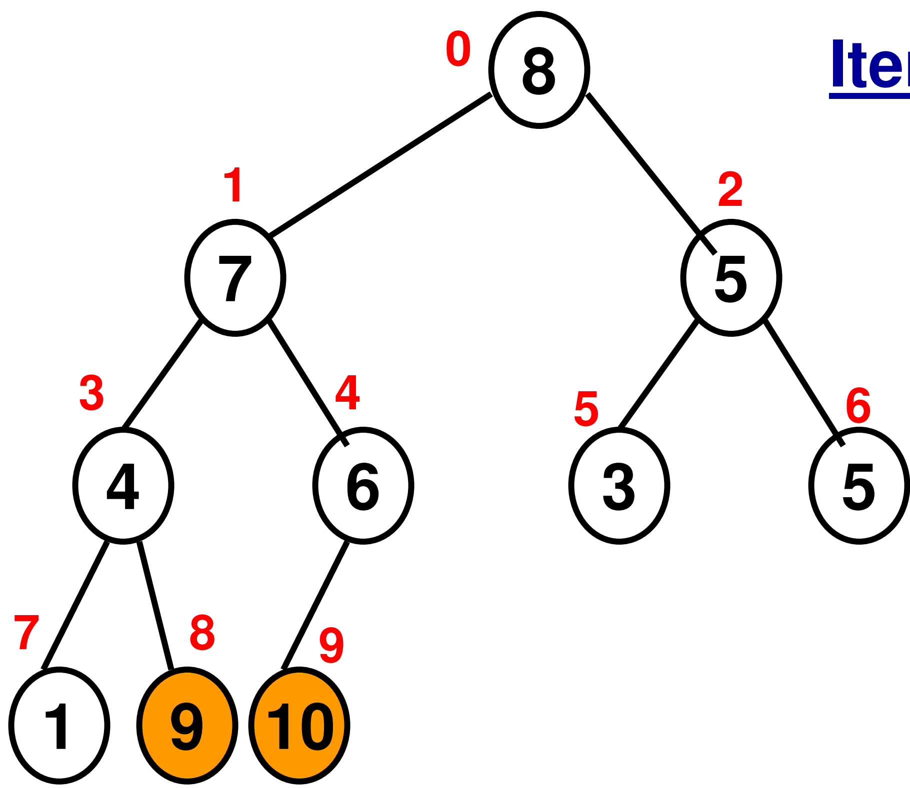
1 8

After Swap

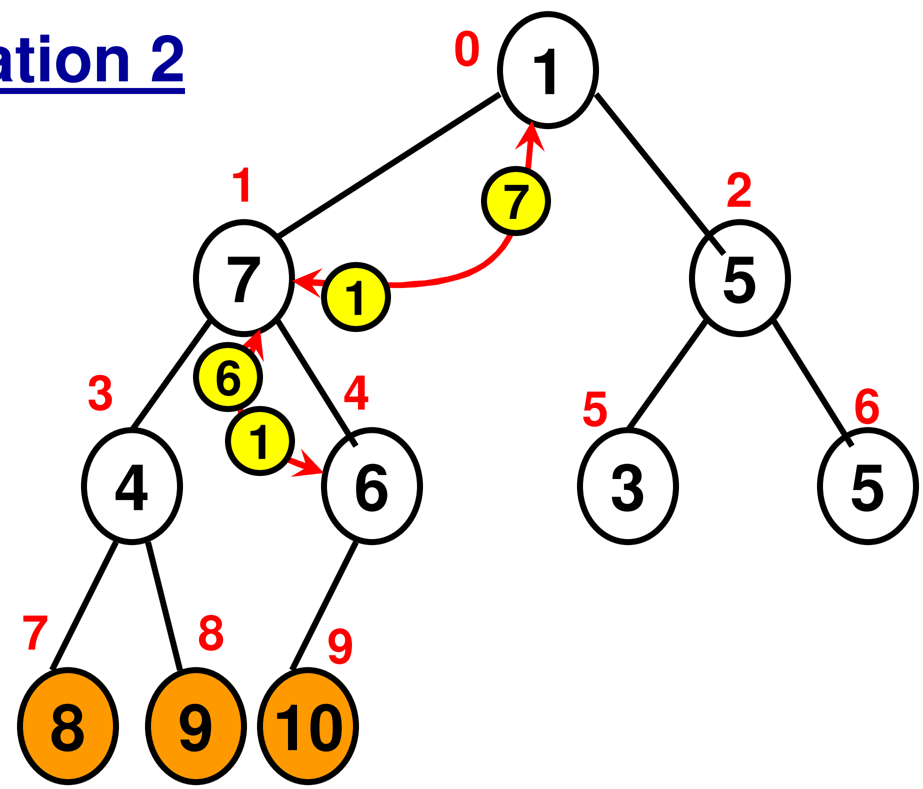
0	1	2	3	4	5	6	7	8	9
1	7	5	4	6	3	5	8	9	10

After Reheapify

0	1	2	3	4	5	6	7	8	9
7	6	5	4	1	3	5	8	9	10



Iteration 2



Max-Heap Version

0	1	2	3	4	5	6	7	8	9
7	6	5	4	1	3	5	8	9	10

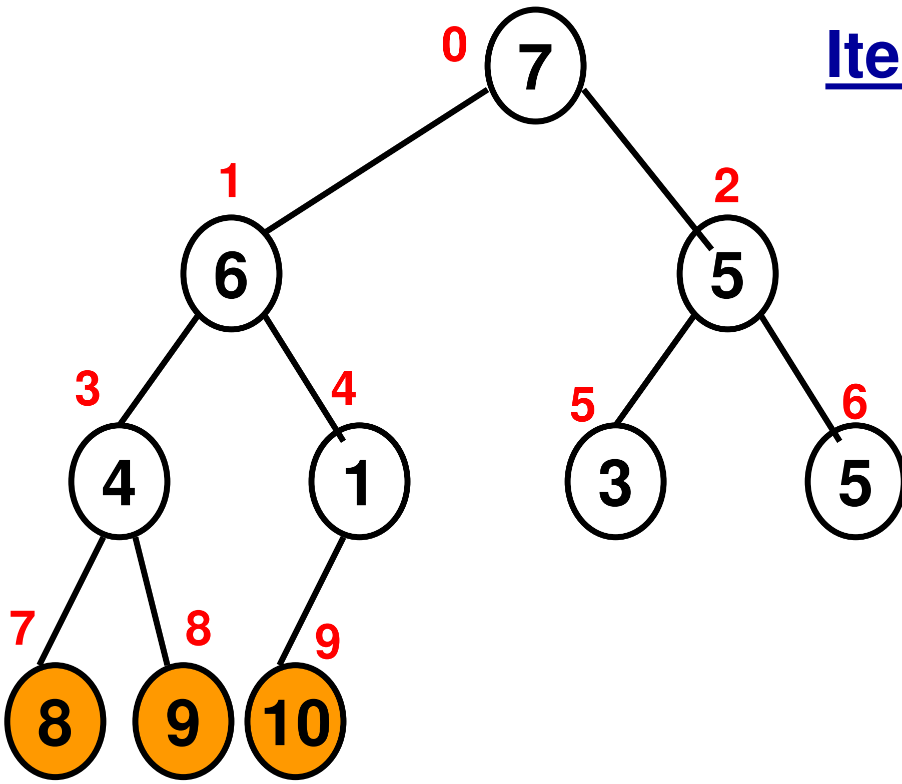
5 7

After Swap

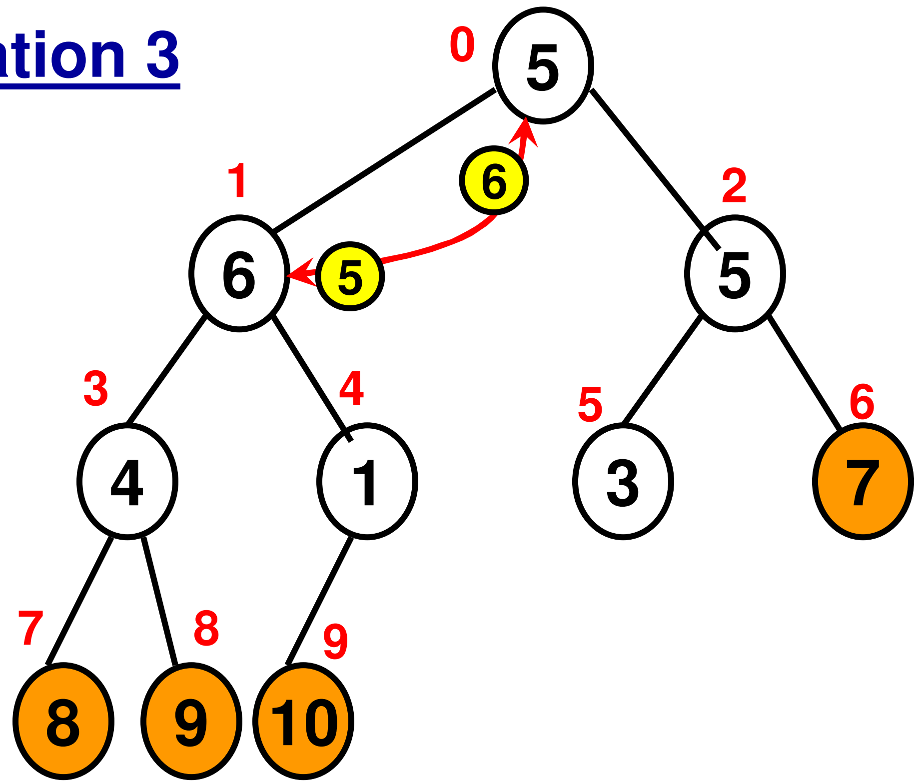
0	1	2	3	4	5	6	7	8	9
5	6	5	4	1	3	7	8	9	10

After Reheapify

0	1	2	3	4	5	6	7	8	9
6	5	5	4	1	3	7	8	9	10



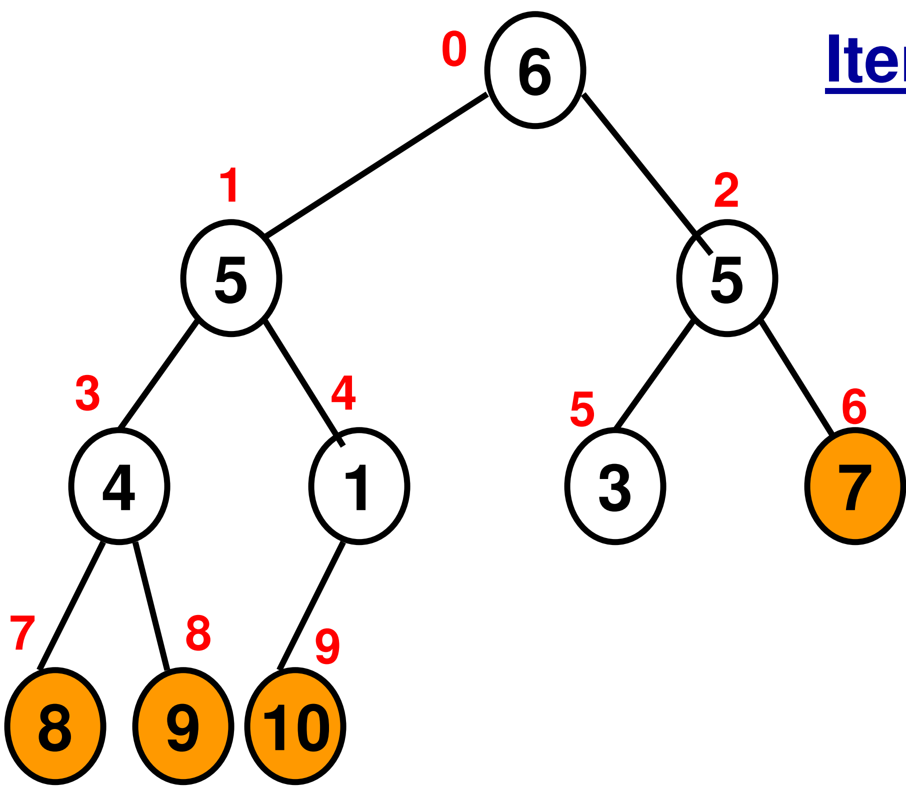
Iteration 3

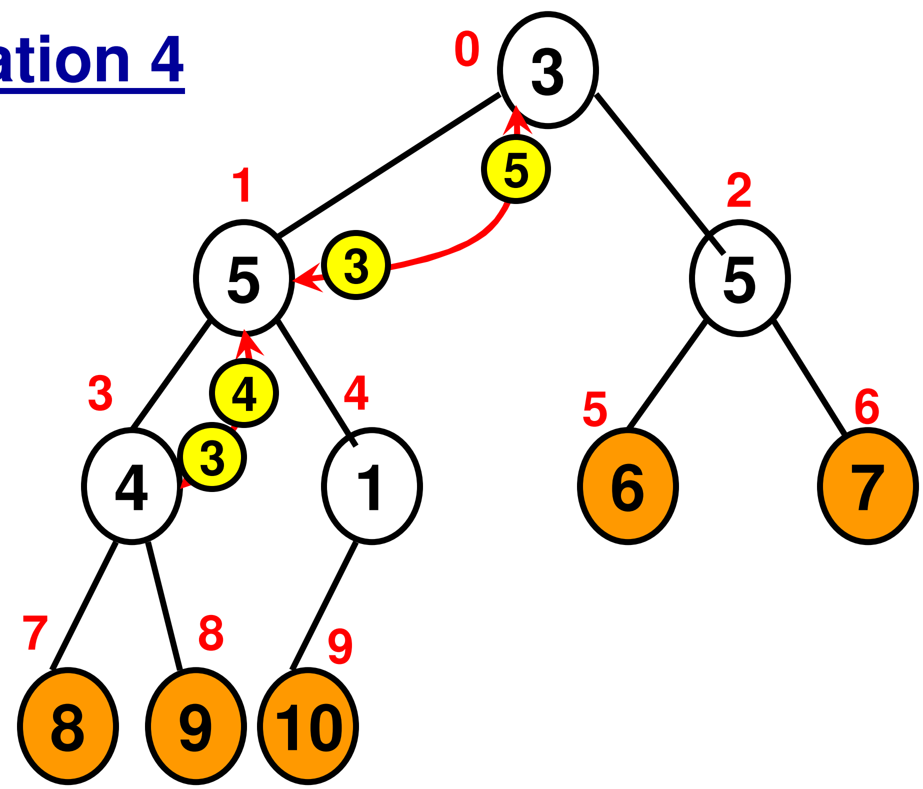


	0	1	2	3	4	5	6	7	8	9
Max-Heap Version	6	5	5	4	1	3	7	8	9	10
After Swap	3	5	5	4	1	6	7	8	9	10
After Reheapify	5	4	5	3	1	6	7	8	9	10

3 6

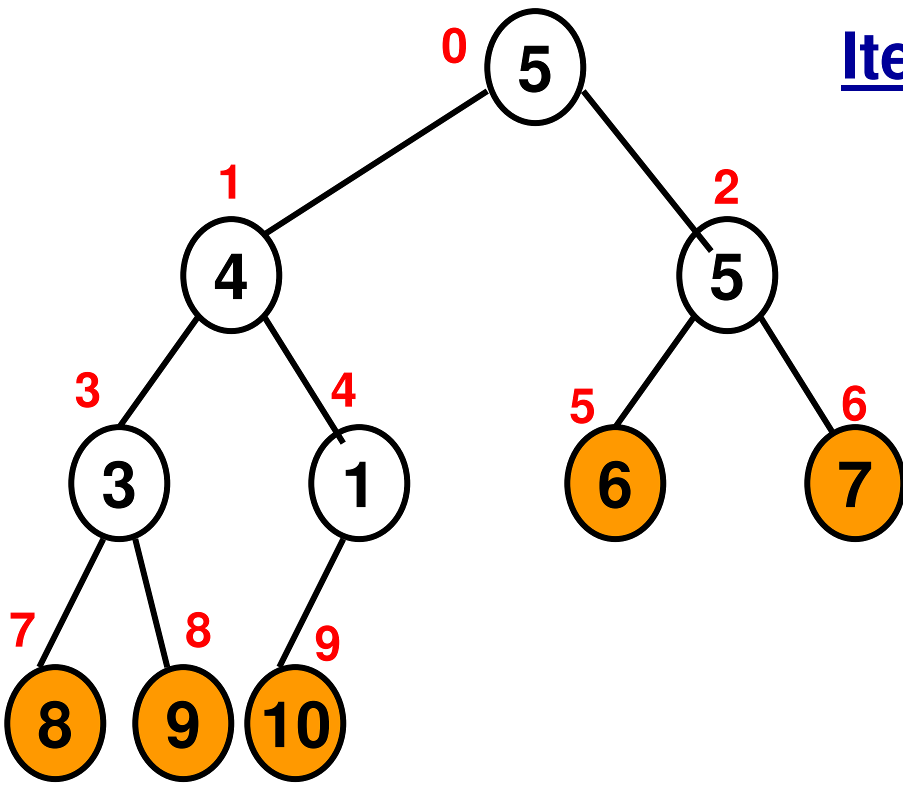
Iteration 4

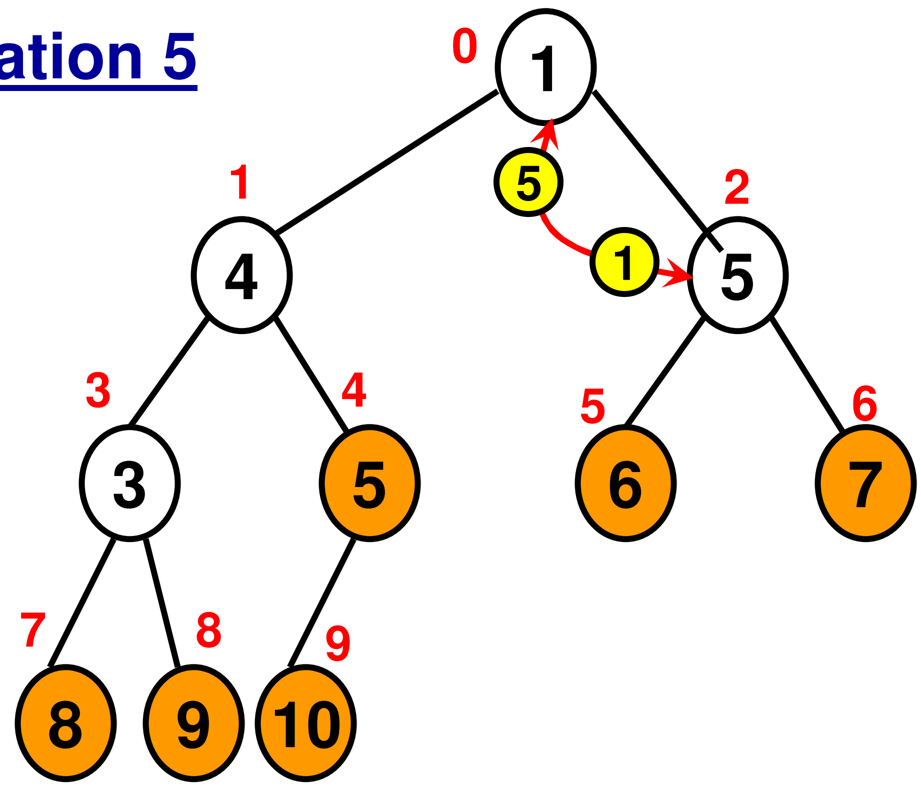




	0	1	2	3	4	5	6	7	8	9
Max-Heap Version	5	4	5	3	1	6	7	8	9	10
After Swap	1	4	5	3	5	6	7	8	9	10
After Reheapify	5	4	1	3	5	6	7	8	9	10

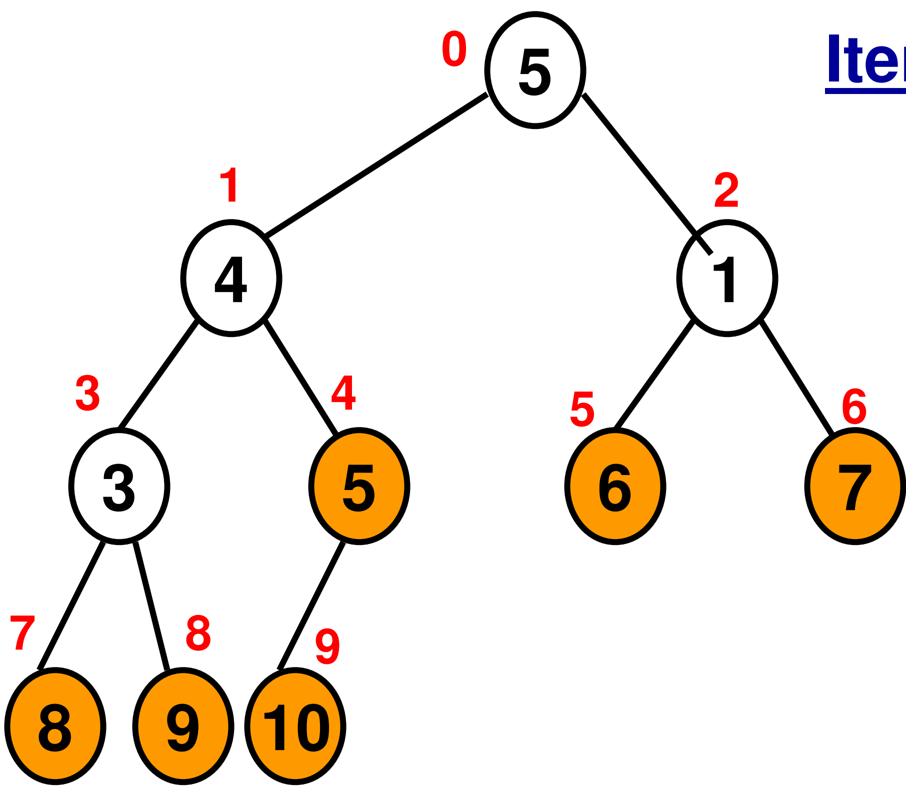
Iteration 5

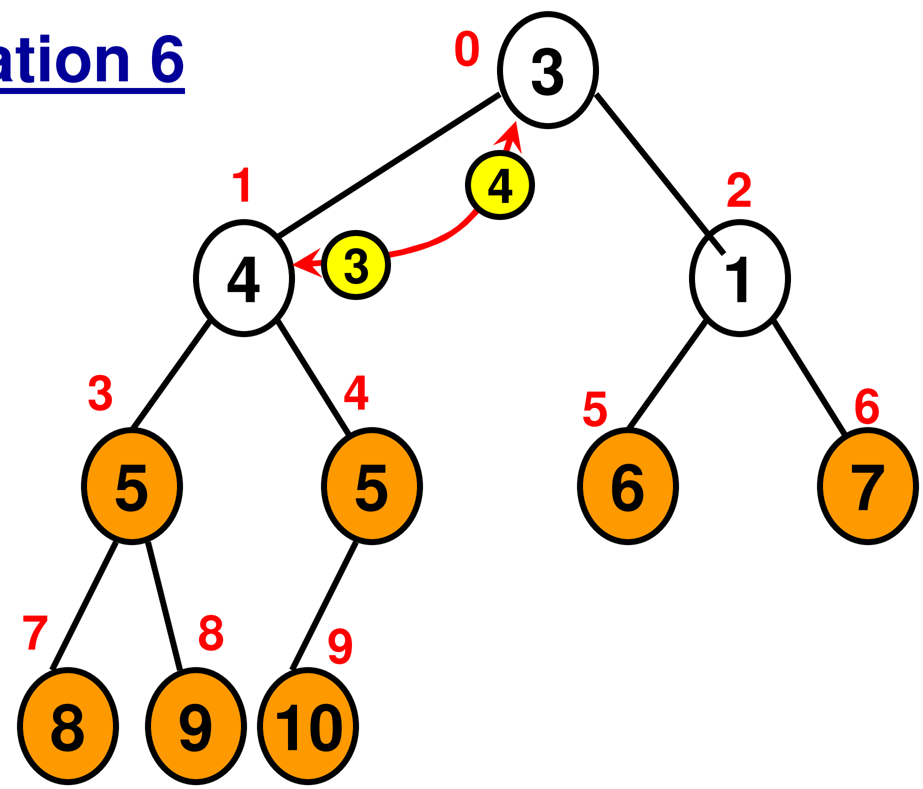




	0	1	2	3	4	5	6	7	8	9
Max-Heap Version	5	4	1	3	5	6	7	8	9	10
After Swap	3	4	1	5	5	6	7	8	9	10
After Reheapify	4	3	1	5	5	6	7	8	9	10

Iteration 6





Max-Heap Version

0	1	2	3	4	5	6	7	8	9
4	3	1	5	5	6	7	8	9	10

1 4

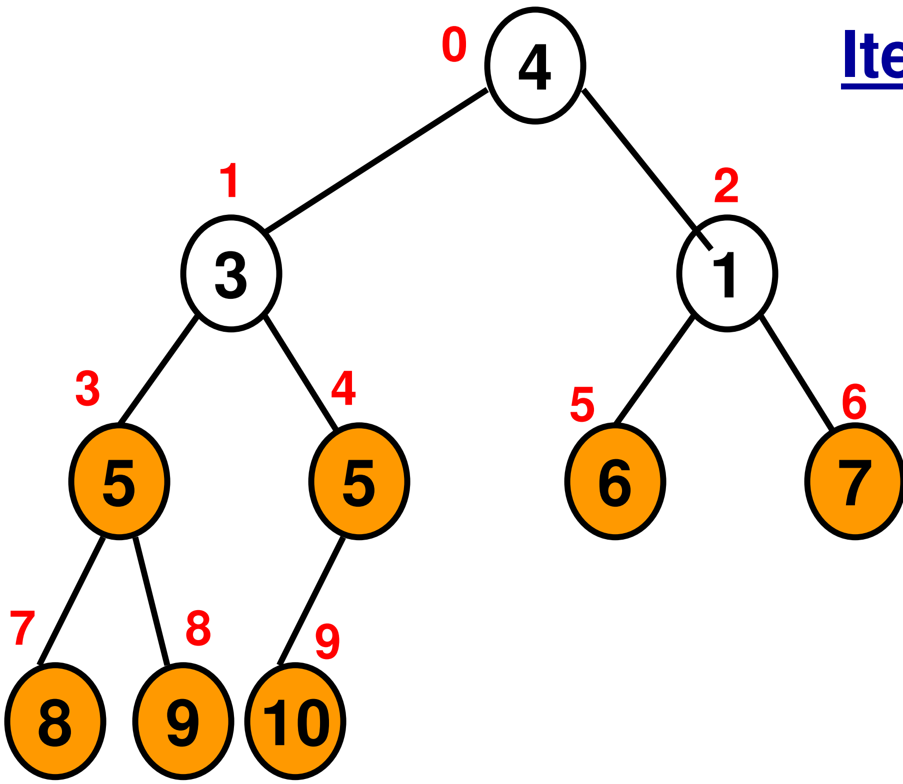
After Swap

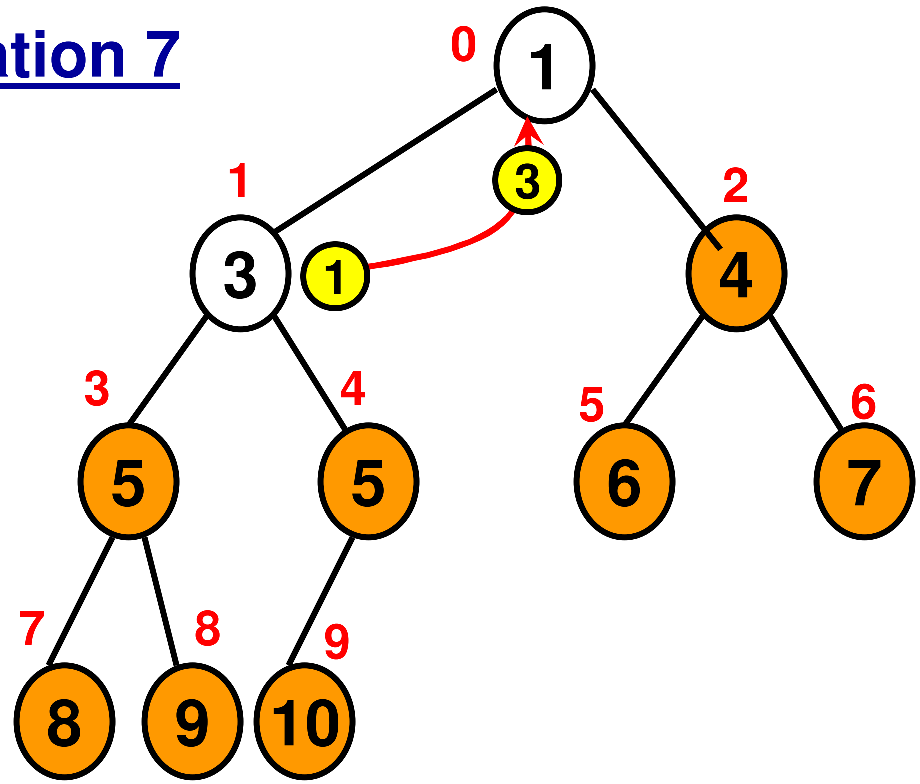
0	1	2	3	4	5	6	7	8	9
1	3	4	5	5	6	7	8	9	10

After Reheapify

0	1	2	3	4	5	6	7	8	9
3	1	4	5	5	6	7	8	9	10

Iteration 7

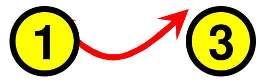




Iteration 8

Max-Heap Version

0	1	2	3	4	5	6	7	8	9
3	1	4	5	5	6	7	8	9	10

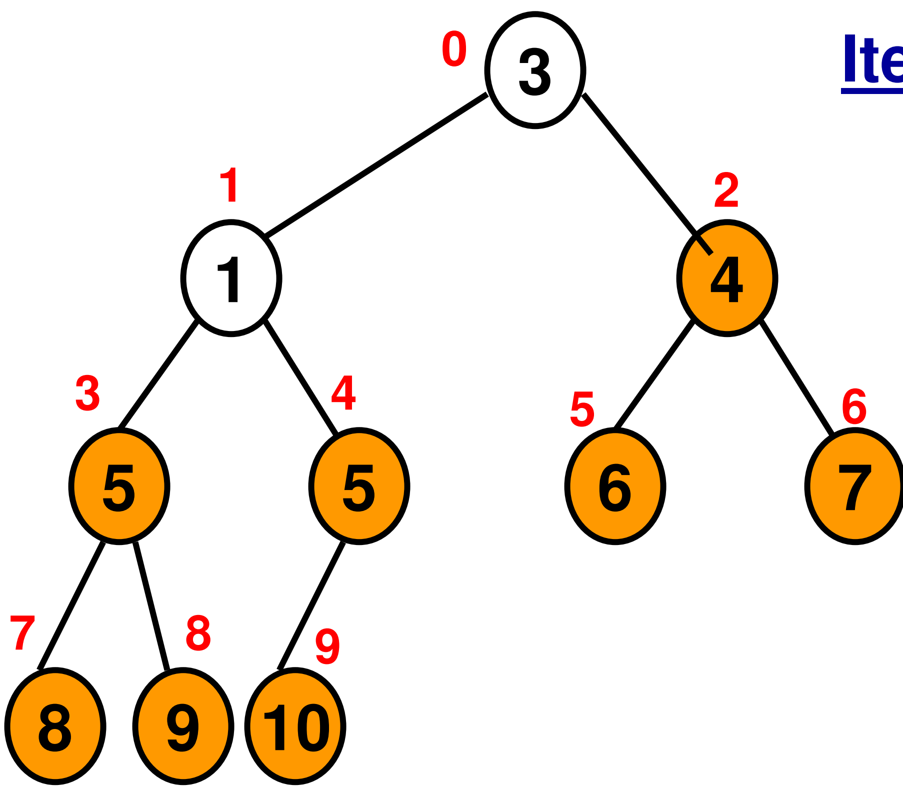


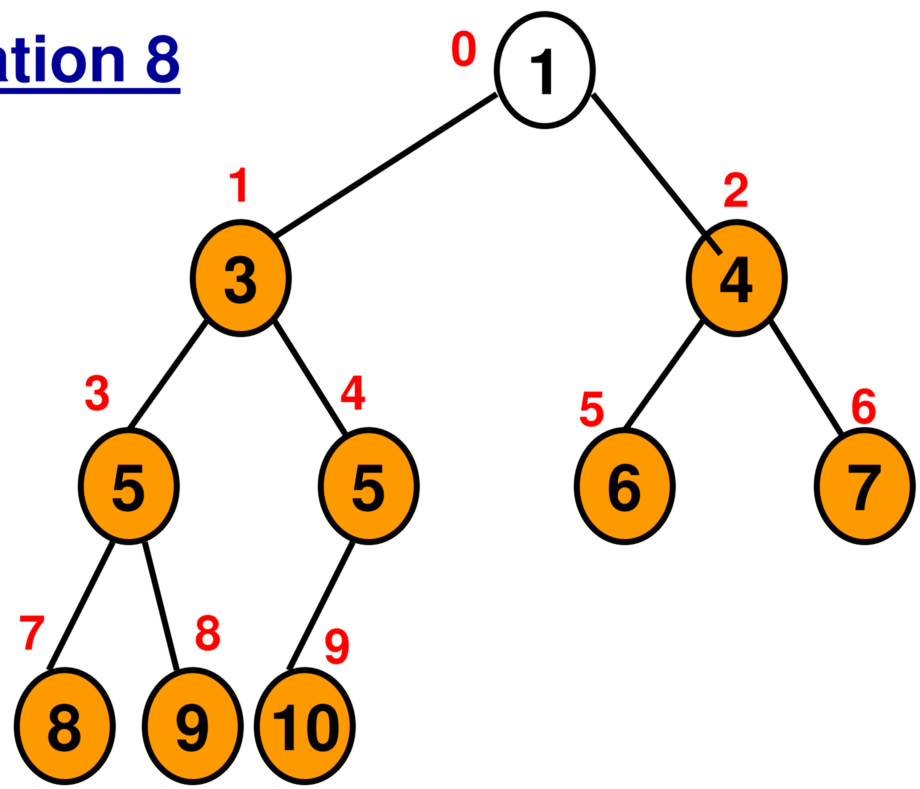
After Swap

0	1	2	3	4	5	6	7	8	9
1	3	4	5	5	6	7	8	9	10

After Reheapify

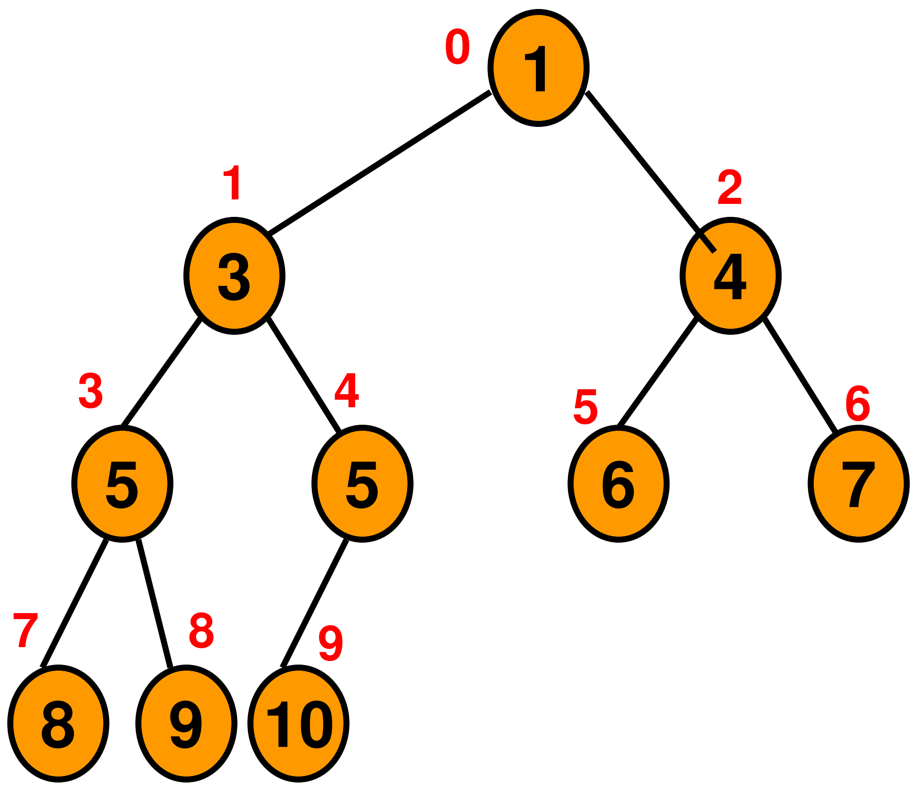
0	1	2	3	4	5	6	7	8	9
1	3	4	5	5	6	7	8	9	10





Heap Sort: Example
Final Sorted Array

0	1	2	3	4	5	6	7	8	9
1	3	4	5	5	6	7	8	9	10



Heap Sort (Code 8.1: C++)

```
for (int iterationIndex = 0; iterationIndex < arraySize; iterationIndex++){

    int temp = array[0];
    array[0] = array[arraySize-1-iterationIndex];
    array[arraySize-1-iterationIndex] = temp;

    rearrangeHeapArray(array, arraySize-1-iterationIndex, 0);

    cout << "Iteration " << iterationIndex << " : ";
    for (int index = 0; index < arraySize; index++)
        cout << array[index] << " ";

    cout << endl;

}
```

Swap the element at the top of the heap with the element at the last index (arraySize-1-iterationIndex) in the active portion of the array

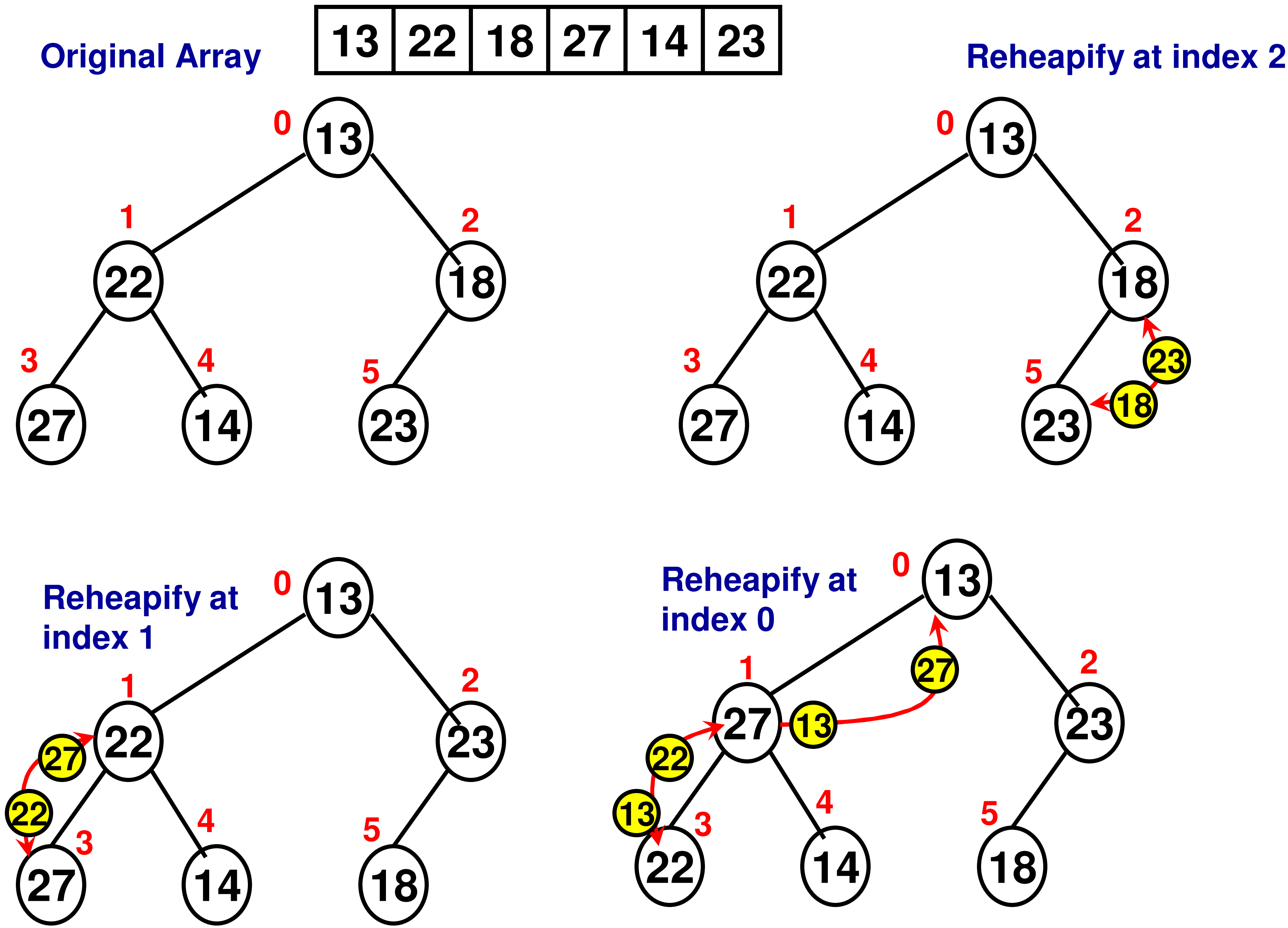
arraySize-1-iterationIndex is also the number of elements in the Unsorted portion of the array (in otherwords, the size of the active portion of the array)

Heap Sort (Code 8.1: Java)

```
for (int iterationIndex = 0; iterationIndex < arraySize; iterationIndex++){

    int temp = array[0];
    array[0] = array[arraySize-1-iterationIndex];
    array[arraySize-1-iterationIndex] = temp;

    rearrangeHeapArray(array, arraySize-1-iterationIndex, 0);

    System.out.print("Iteration " + iterationIndex + " : ");
    for (int index = 0; index < arraySize; index++)
        System.out.print(array[index] + " ");

    System.out.println();

}
```

Swap the element at the top of the heap with the element at the last index (arraySize-1-iterationIndex) in the active portion of the array

arraySize-1-iterationIndex is also the number of elements in the Unsorted portion of the array (in otherwords, the size of the active portion of the array)


Original Array
13 22 18 27 14 23
Reheapify at index 2
Reheapify at index 1
Reheapify at index 0

Heap Sort: Example 2

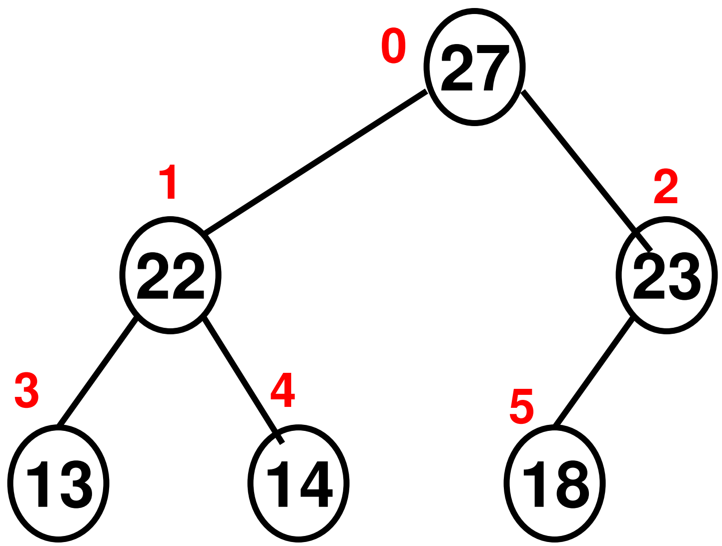
Original Array

13	22	18	27	14	23

Max-Heap Version

0	1	2	3	4	5
27	22	23	13	14	18

Max-Heap



Max-Heap Version

0	1	2	3	4	5
27	22	23	13	14	18

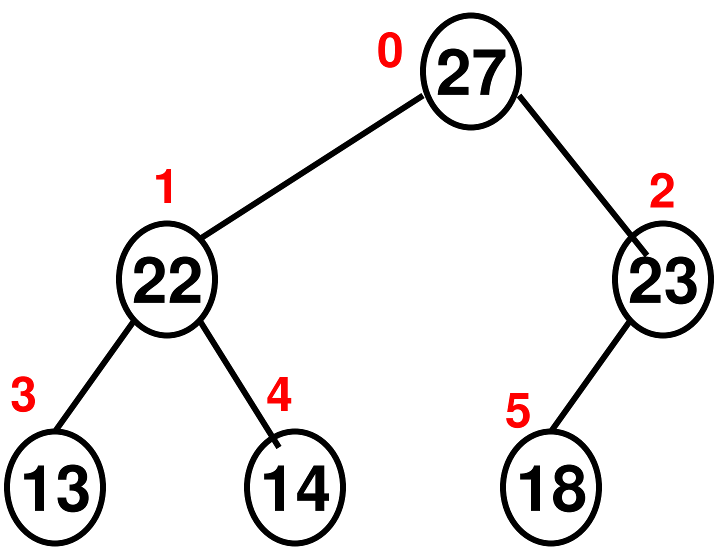
After Swap

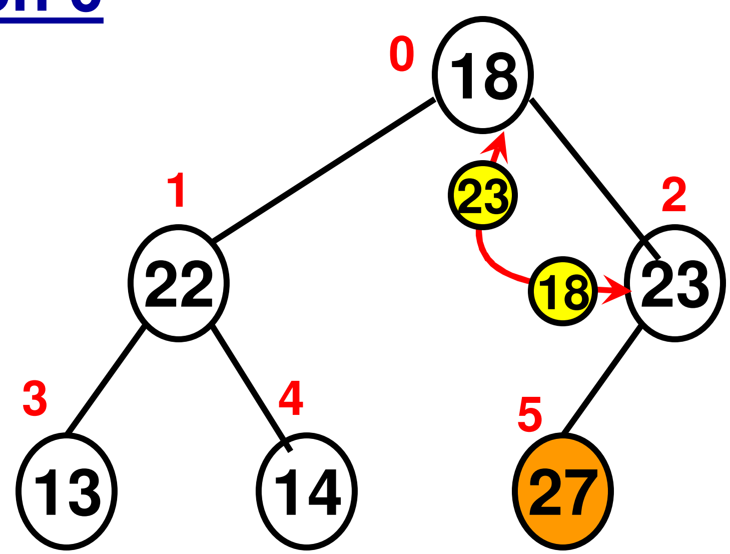
0	1	2	3	4	5
18	22	23	13	14	27

After Reheapify

0	1	2	3	4	5
23	22	18	13	14	27

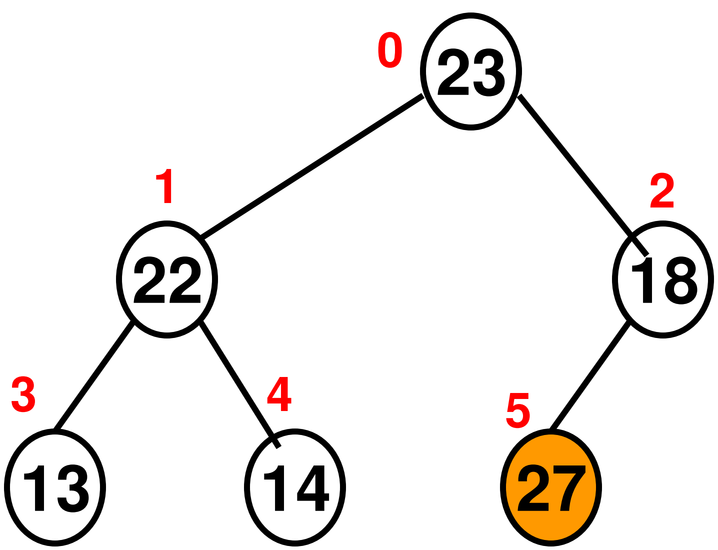
Iteration 0

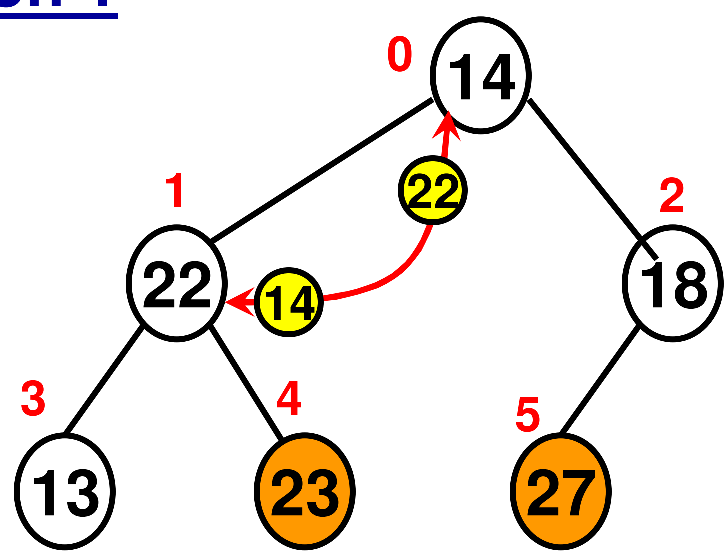




	0	1	2	3	4	5
Max-Heap Version	23	22	18	13	14	27
After Swap	14	22	18	13	23	27
After Reheapify	22	14	18	13	23	27

Iteration 1





Max-Heap Version

0	1	2	3	4	5
22	14	18	13	23	27

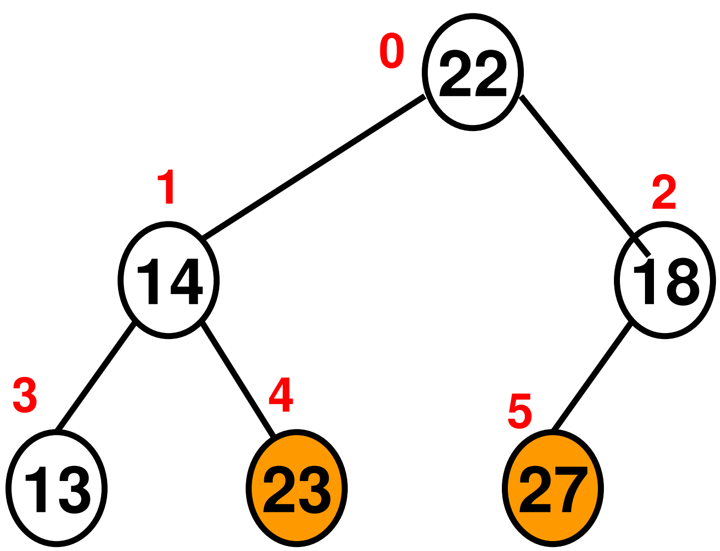
After Swap

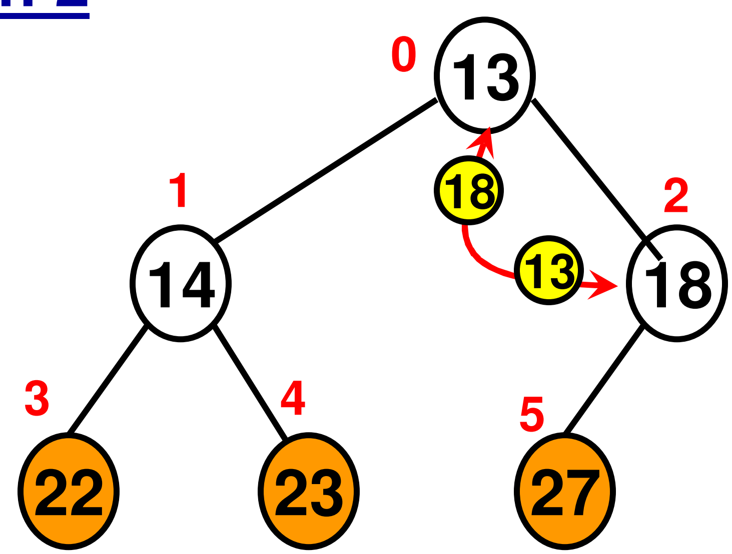
0	1	2	3	4	5
13	14	18	22	23	27

After Reheapify

0	1	2	3	4	5
18	14	13	22	23	27

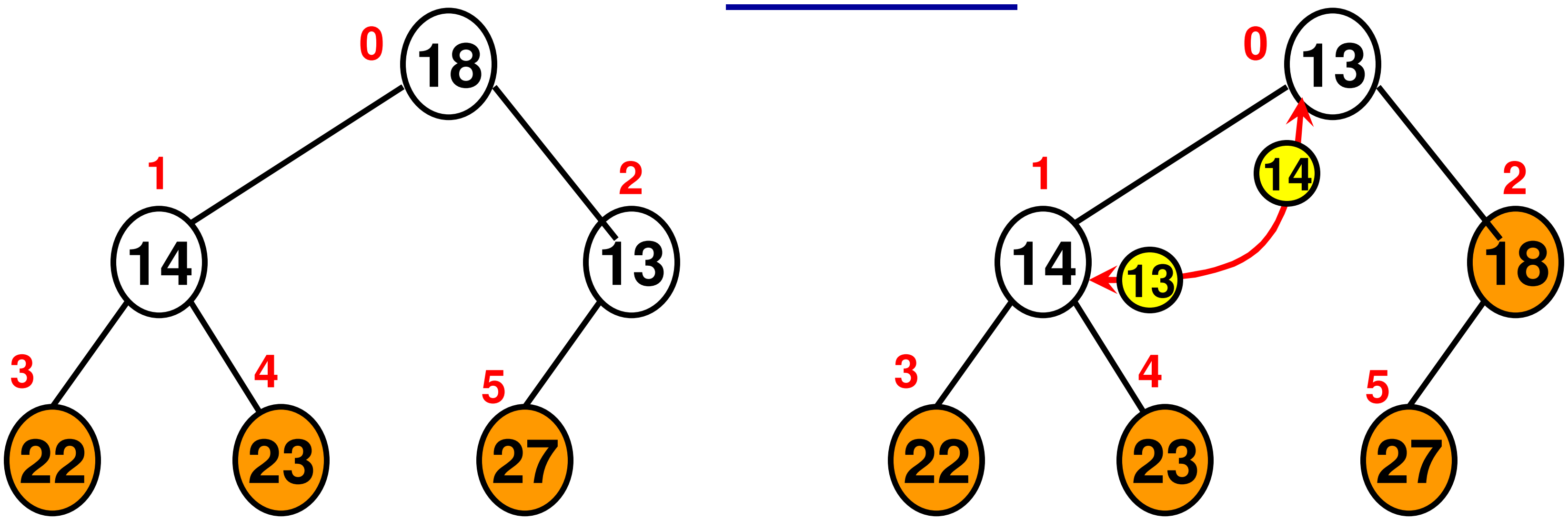
Iteration 2





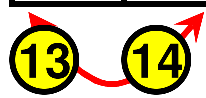
	0	1	2	3	4	5
Max-Heap Version	18	14	13	22	23	27
After Swap	13	14	18	22	23	27
After Reheapify	14	13	18	22	23	27

Iteration 3



Max-Heap Version

0	1	2	3	4	5
14	13	18	22	23	27



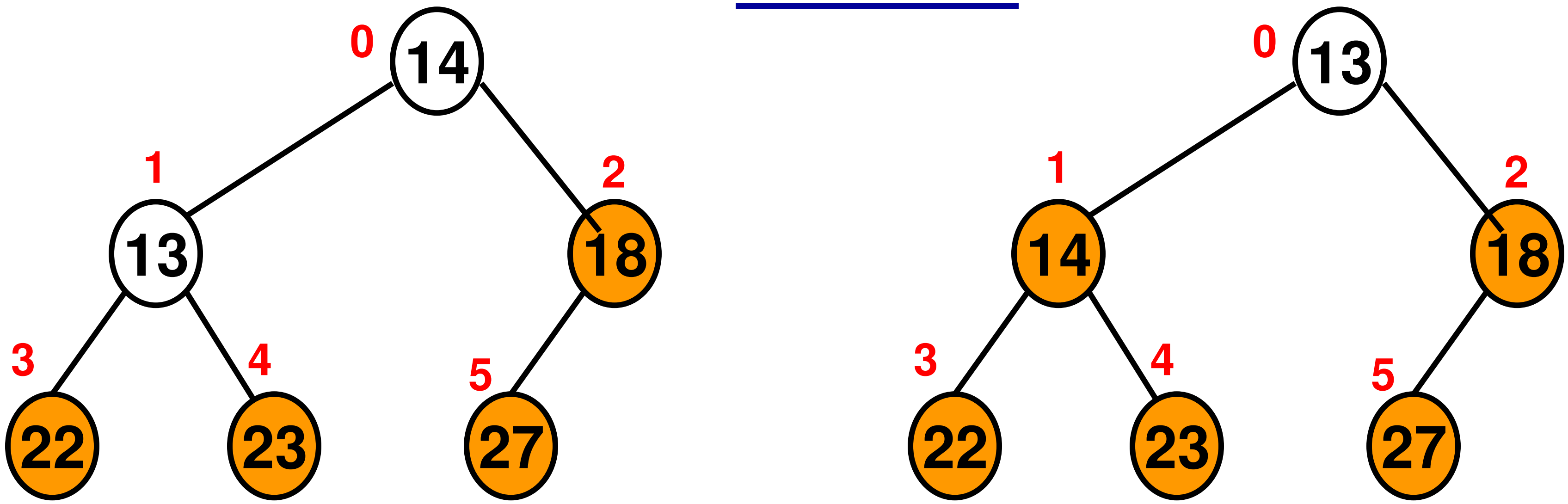
After Swap

0	1	2	3	4	5
13	14	18	22	23	27

After Reheapify

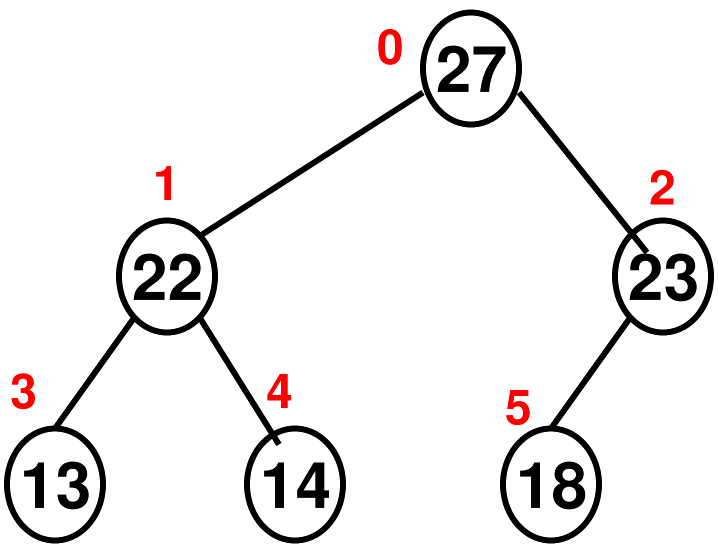
0	1	2	3	4	5
13	14	18	22	23	27

Iteration 4

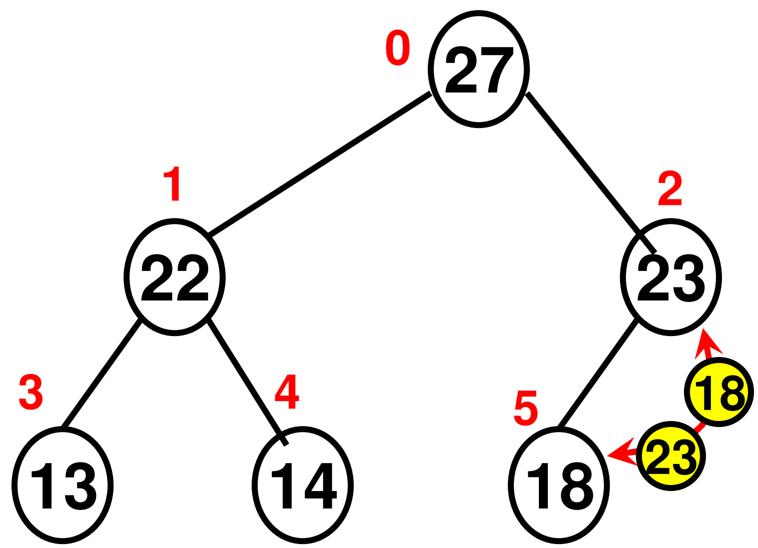


Max Heap to Min Heap (direct transformation): Example 1

- Given a Max Heap, reheapify every internal node to make sure the data at the internal node is lower than or equal to the data of its immediate child nodes.
- This would take O(n) time (like the transformation of an arbitrary essentially complete binary tree to max heap or min heap).

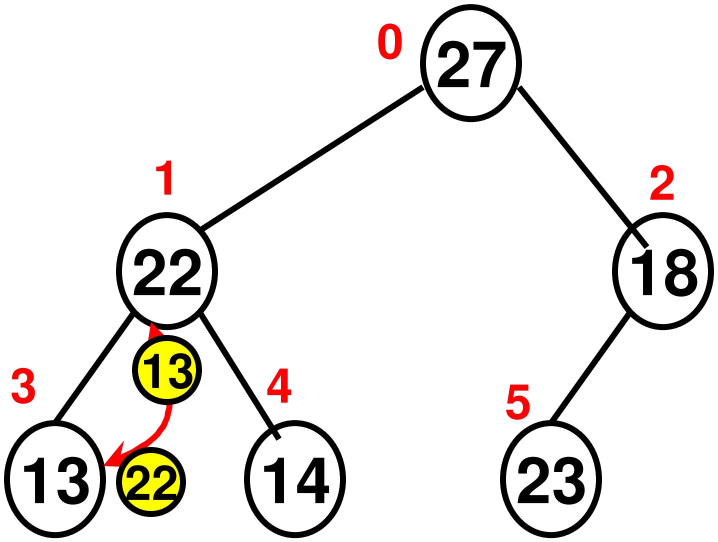


Given Max Heap

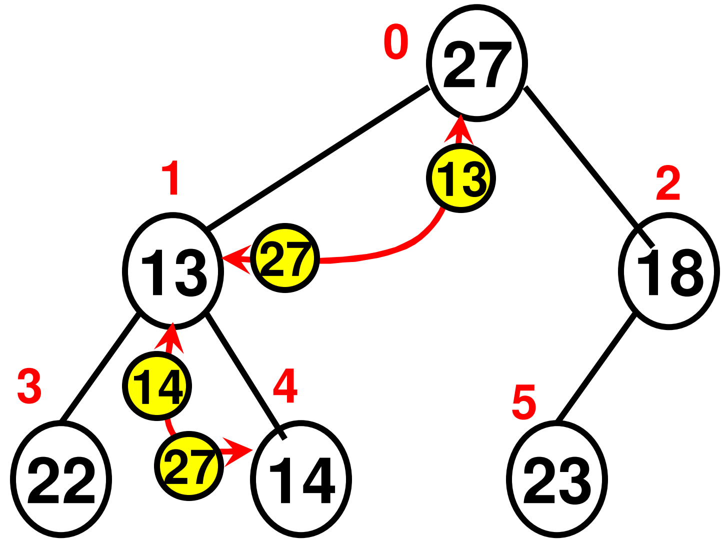


Reheapify at index 2

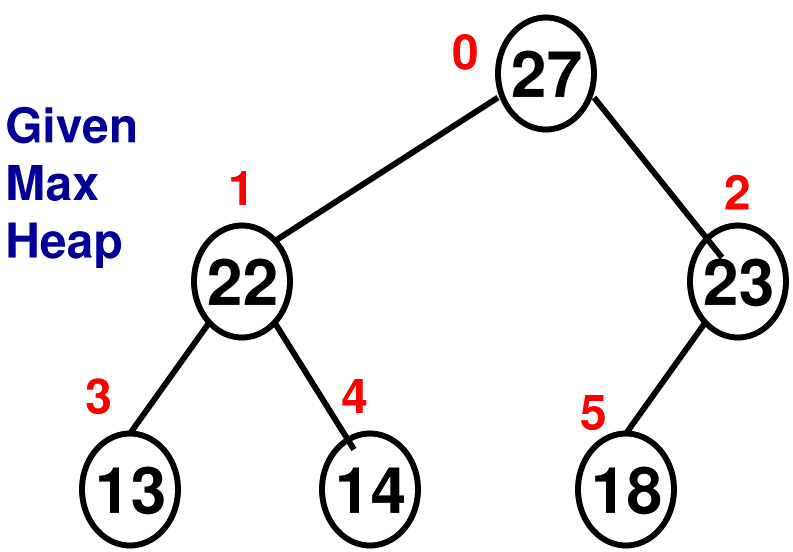
Max Heap to Min Heap: Example 1

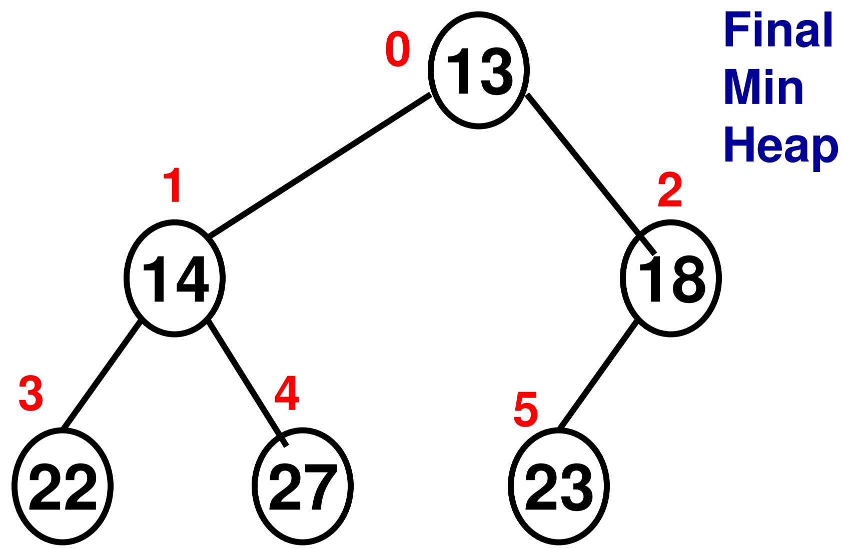


Reheapify at index 1

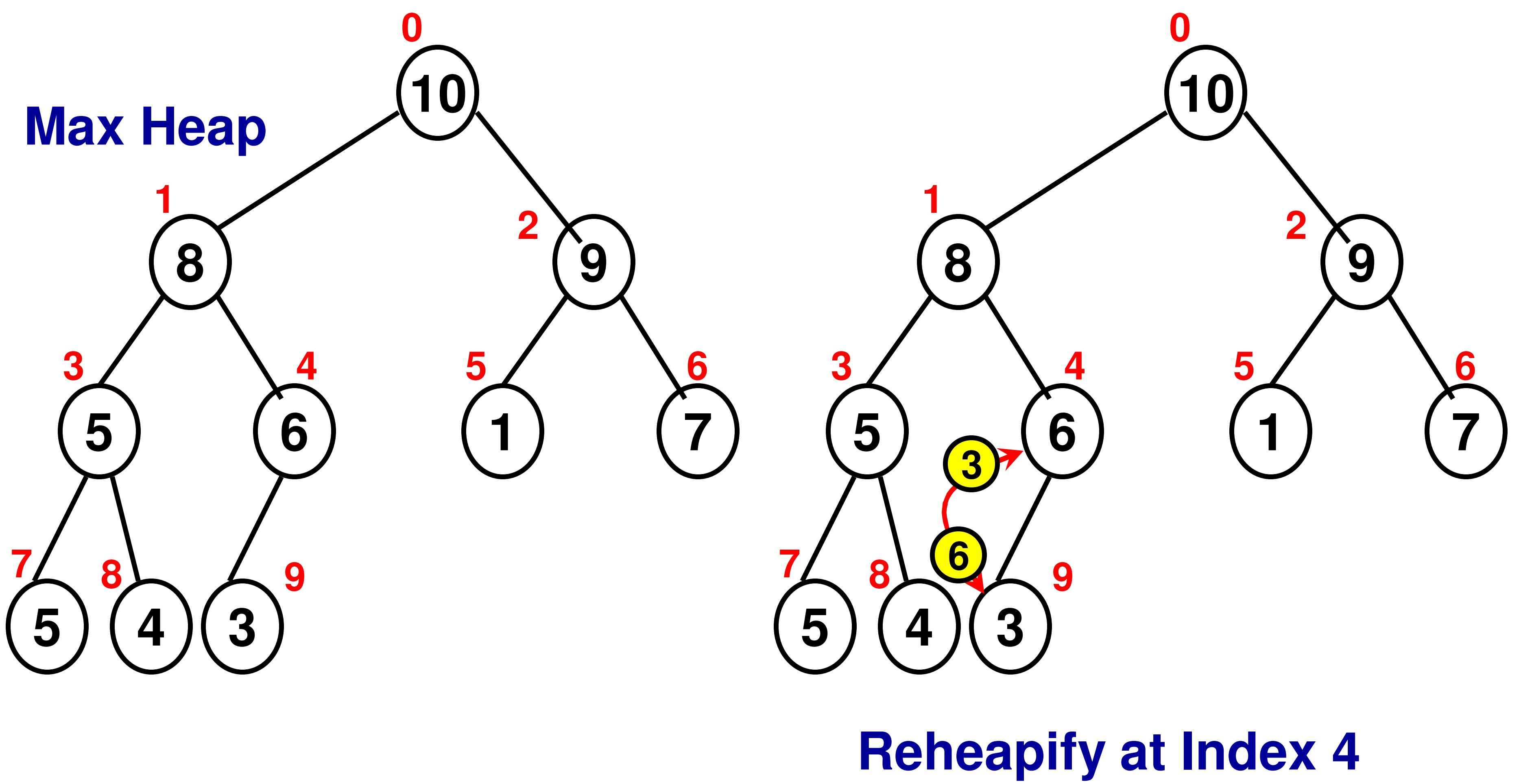


Reheapify at index 0

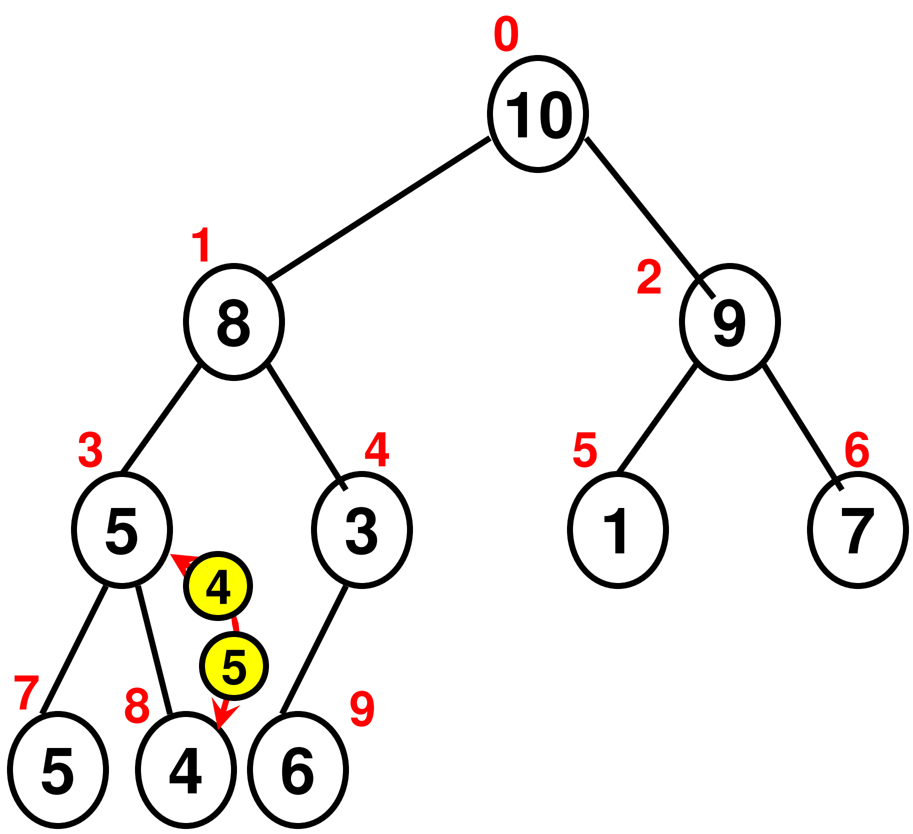




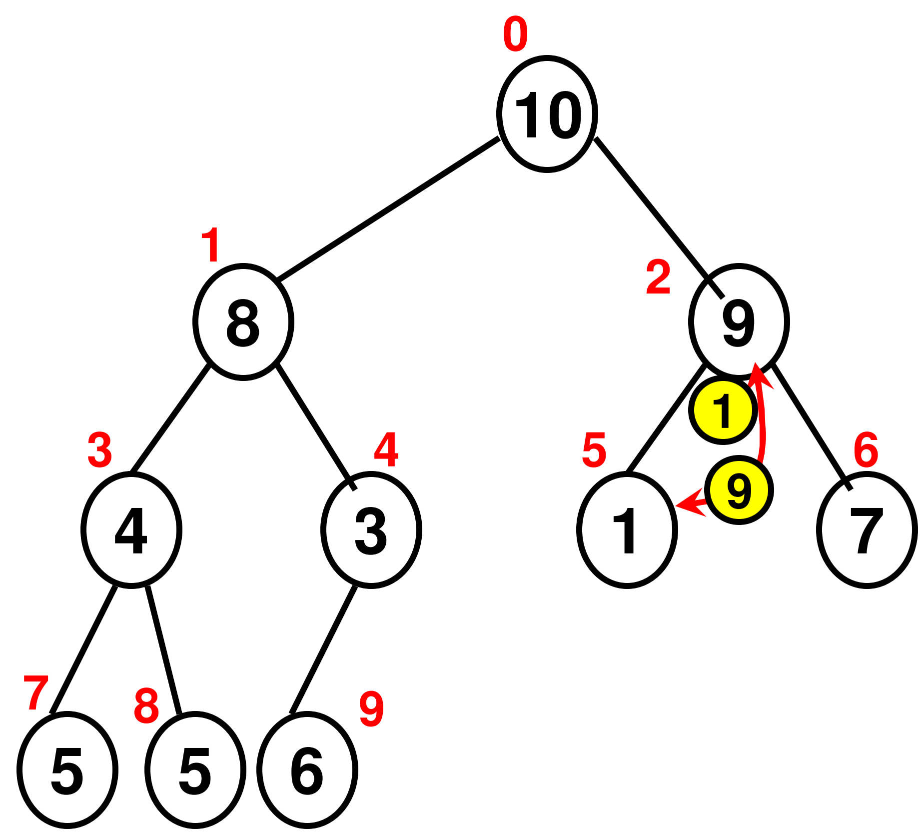
Max Heap to Min Heap: Example 2



Max Heap to Min Heap: Example 2

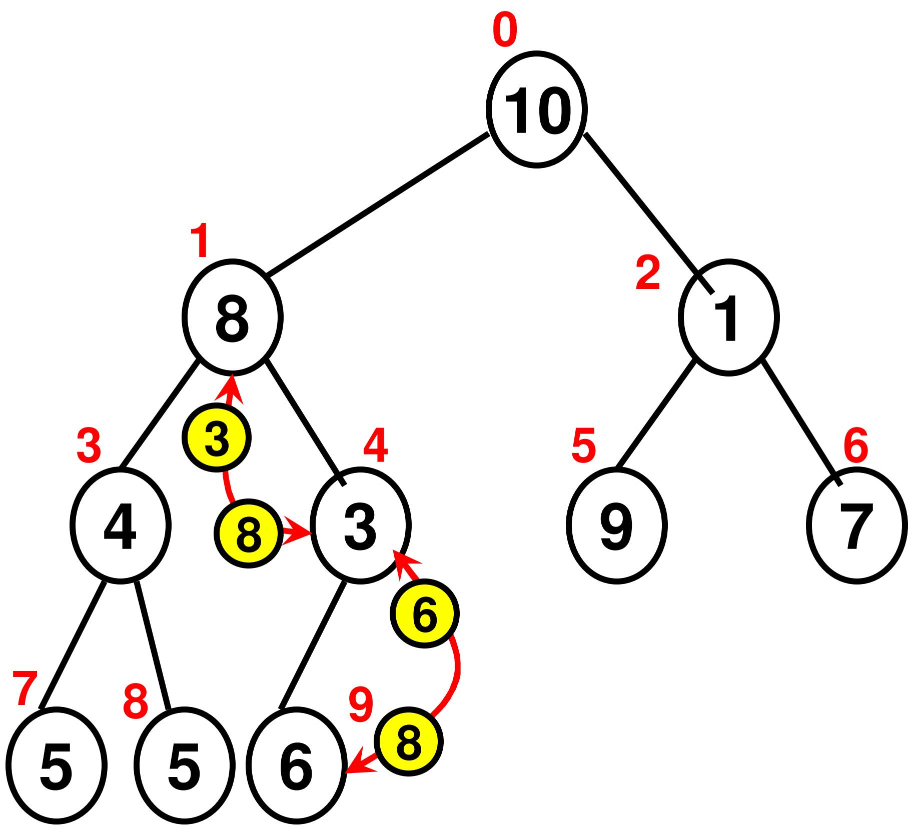


Reheapify at Index 3

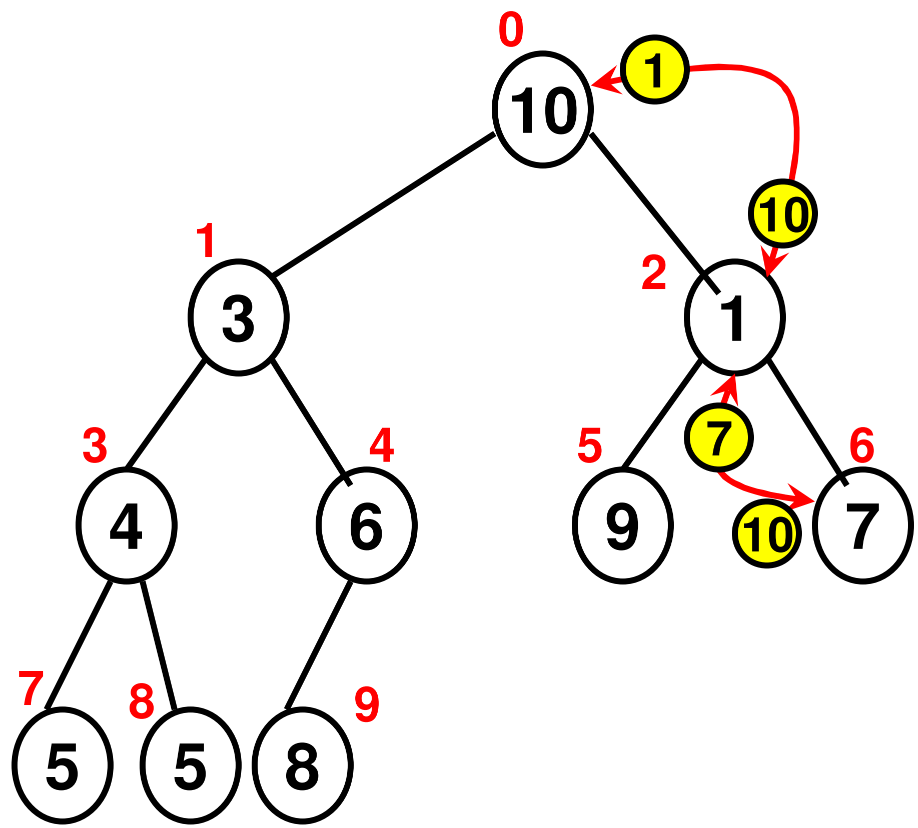


Reheapify at Index 2

Max Heap to Min Heap: Example 2

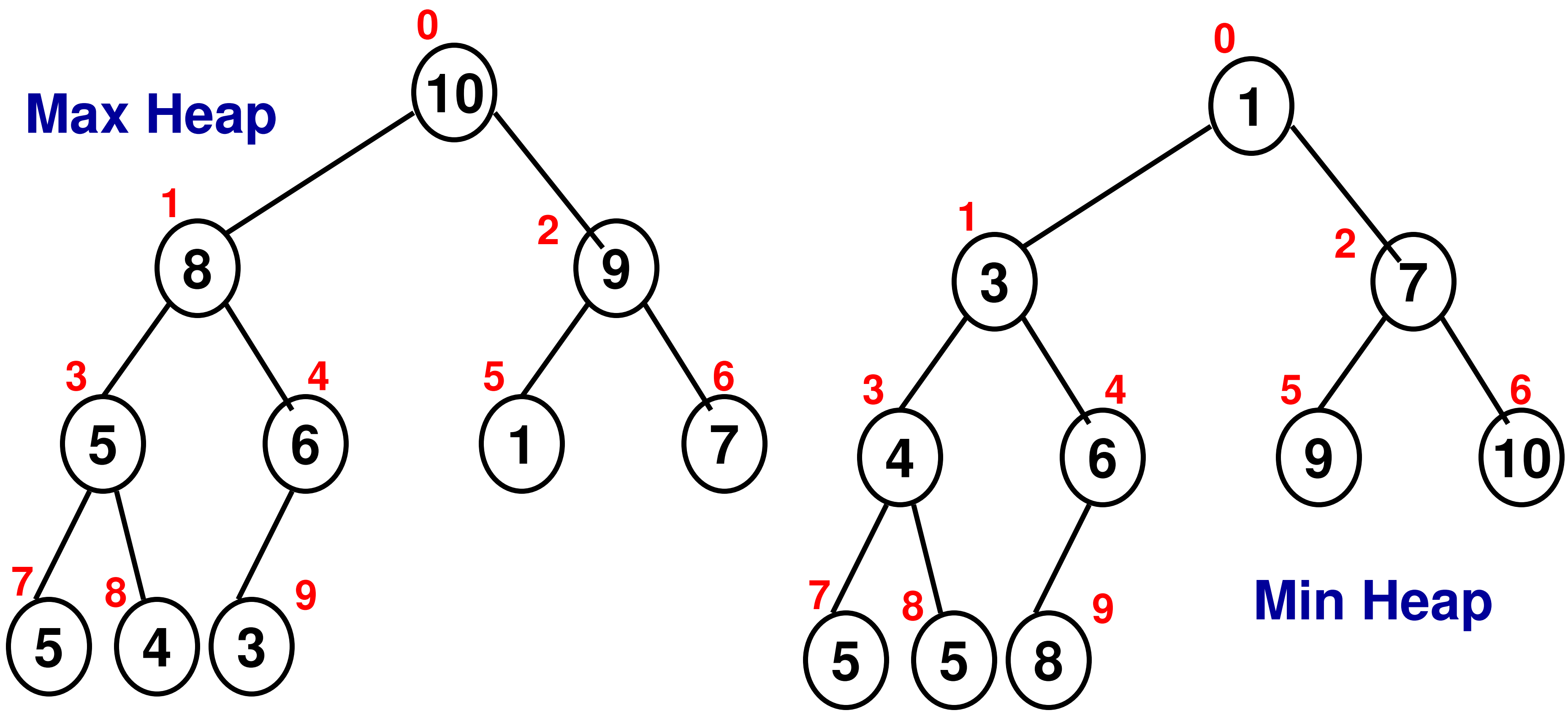


Reheapify at Index 1



Reheapify at Index 0

Max Heap to Min Heap: Example 2



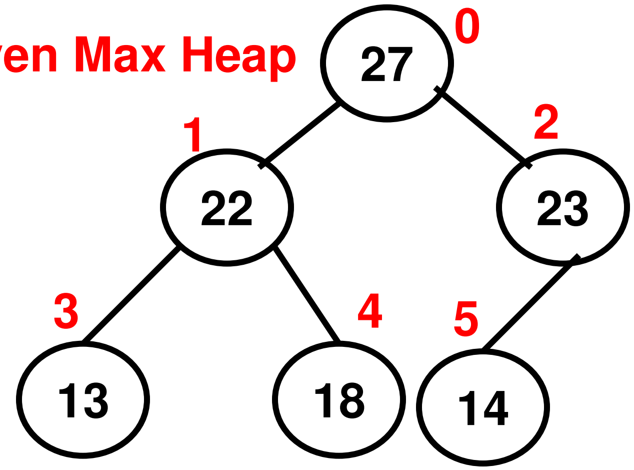
Note that in a Min Heap directly obtained via a sequence of Reheapify operations of the Max Heap, we cannot guarantee that for any internal node: the data of the nodes in its left sub tree are lower than the data for the nodes in its right sub tree. In the above example, we observe the left sub tree of the root node has node 9 with data ‘8’ that is larger than data ‘7’ of node 2 in the right sub tree.

Max Heap to a Min Heap (via a Binary Search Tree)

- **Max Heap to BST**
- **Step 0: Set up the node indices in the top-down, left-right fashion.**
- **Step 1: Do an inorder traversal of the Max Heap and obtain the sequence of node indices and their corresponding data**
- **Step 2: Sort the data got from the inorder traversal**
- **Step 3: Replace the node indices with the data in the sorted order**
- **BST to Min Heap**
- Perform an inorder traversal of the BST and create an array of the sorted integers of the data corresponding to the nodes in the BST.
- Perform a preorder traversal of the BST. While performing the preorder traversal, replace the data at each node visited with the values of the inorder array. The resulting binary tree is a min heap.
- **Since the min heap is generated from a BST, the min heap has the following property (need not be observed when directly obtained from a max heap):**
  - **For any internal node: the data of all the nodes in the left sub tree are less than or equal to the data of all the nodes in the right sub tree.**

Max Heap to BST: Example 1

Given Max Heap



Inorder traversal of node indices/data

3	**1**	**4**	**0**	**5**	**2**
13	22	18	27	14	23

Sorted order of data

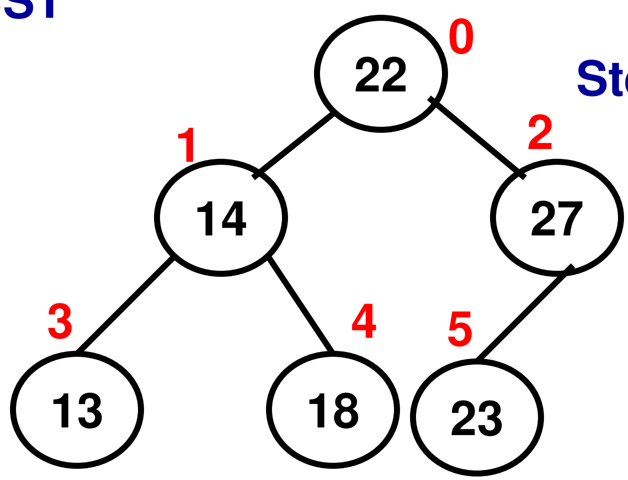
13 14 18 22 23 27

Step 1: Do an inorder traversal of the Max Heap and obtain the sequence of node indices and their corresponding data

Step 2: Sort the data got from the inorder traversal

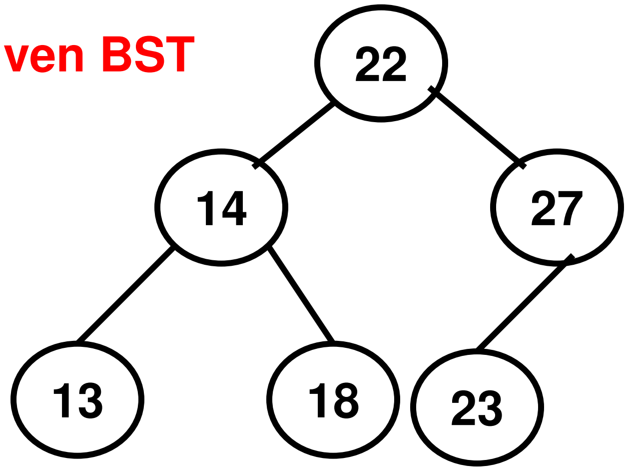
Step 3: Replace the node indices with the data in the sorted order

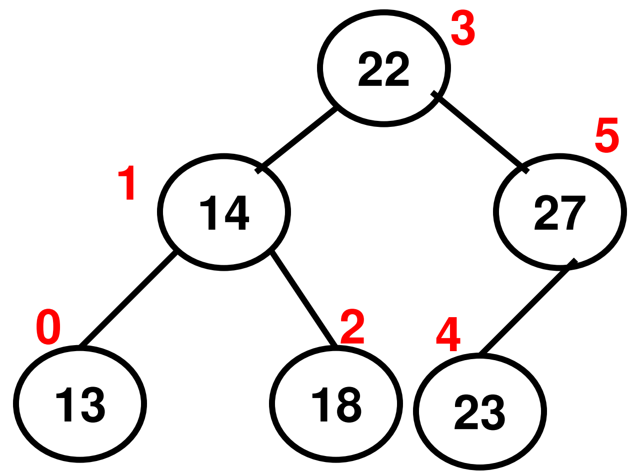
BST



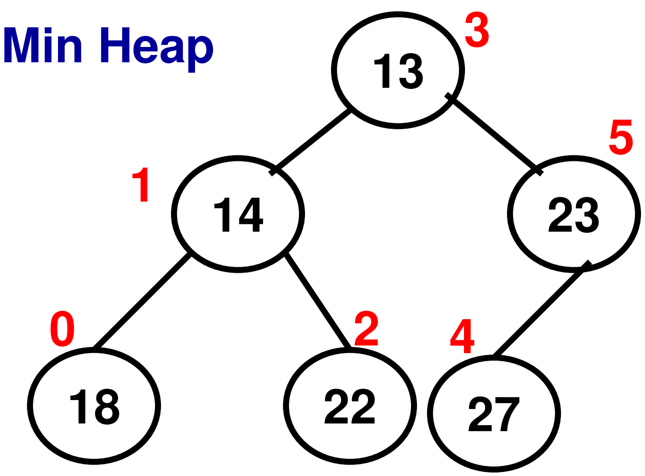
BST to Min Heap: Example 1

Given BST





Min Heap



Inorder-based node indices

0	1	2	3	4	5
13	14	18	22	23	27

Preorder Listing of the node indices

3 1 0 2 5 4

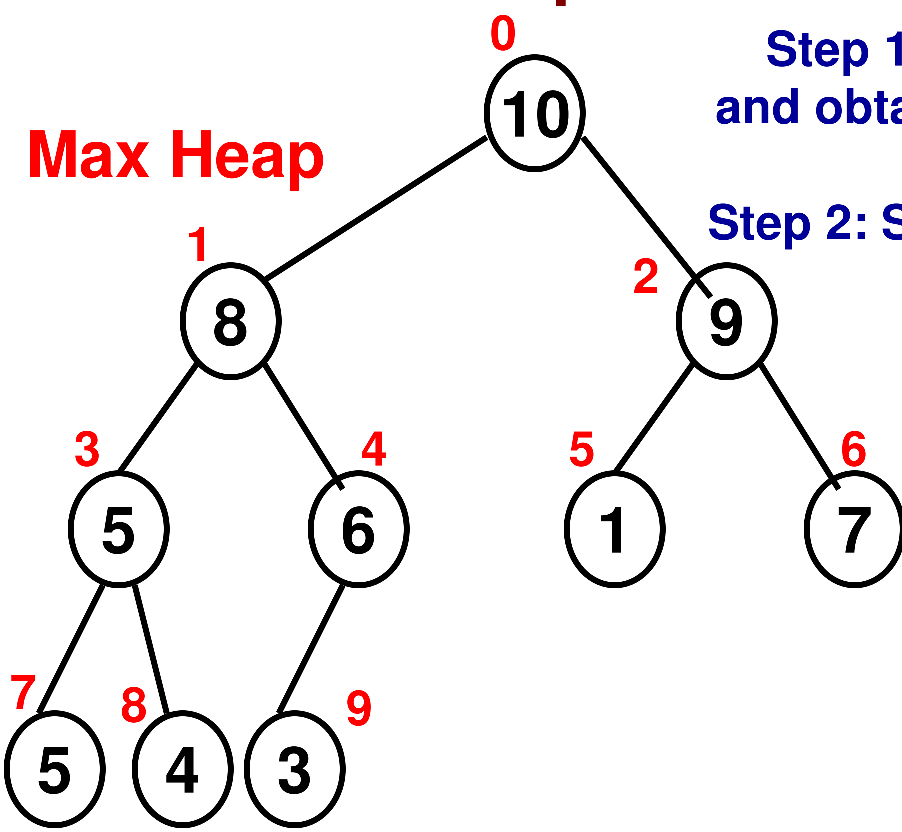
Min heap

3	1	0	2	5	4
13	14	18	22	23	27

Max Heap to a BST: Example 2

Step 1: Do an inorder traversal of the Max Heap and obtain the sequence of node indices and their corresponding data
Step 2: Sort the data got from the inorder traversal
Step 3: Replace the node indices with the data in the sorted order

Max Heap



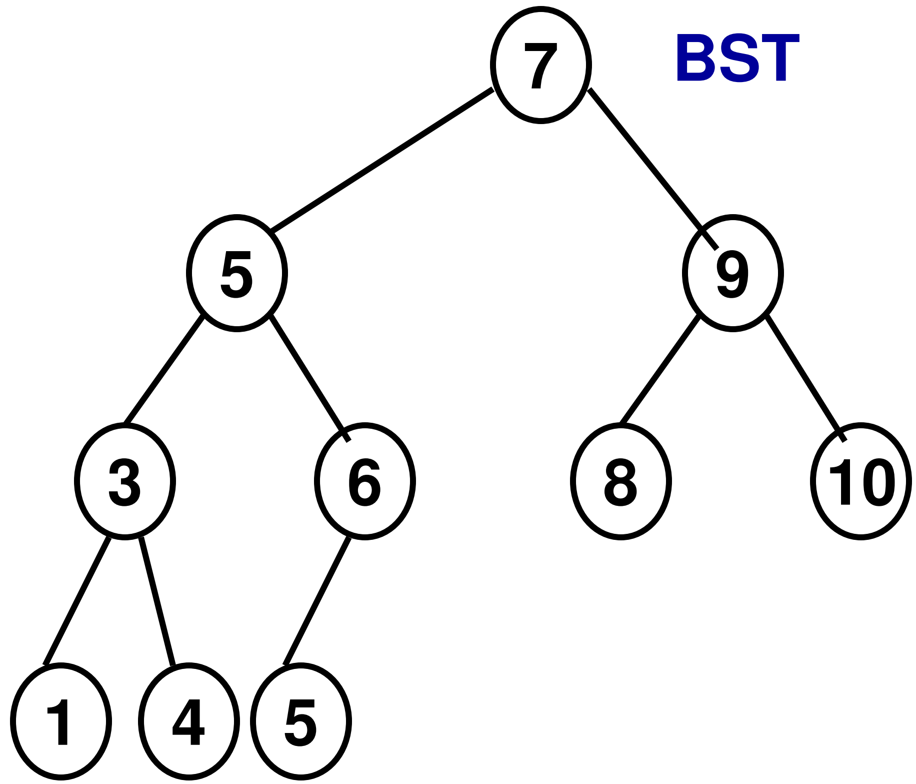
Inorder traversal of node indices/data

7	**3**	**8**	**1**	**9**	**4**	**0**	**5**	**2**	**6**
5	5	4	8	3	6	10	1	9	7

Sorted order of data

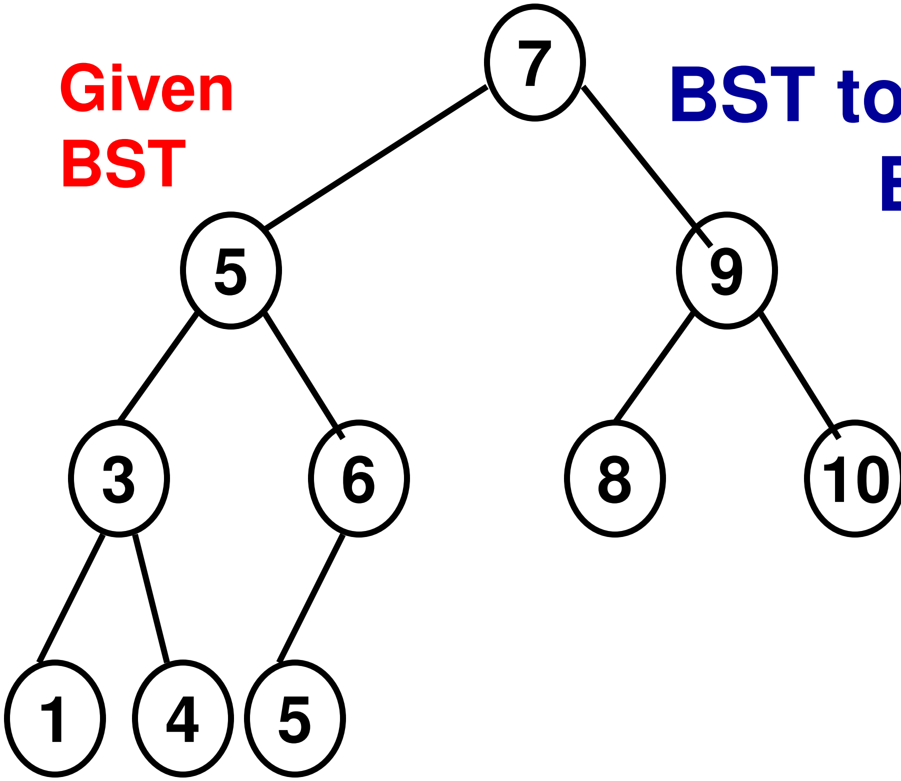
1 3 4 5 5 6 7 8 9 10

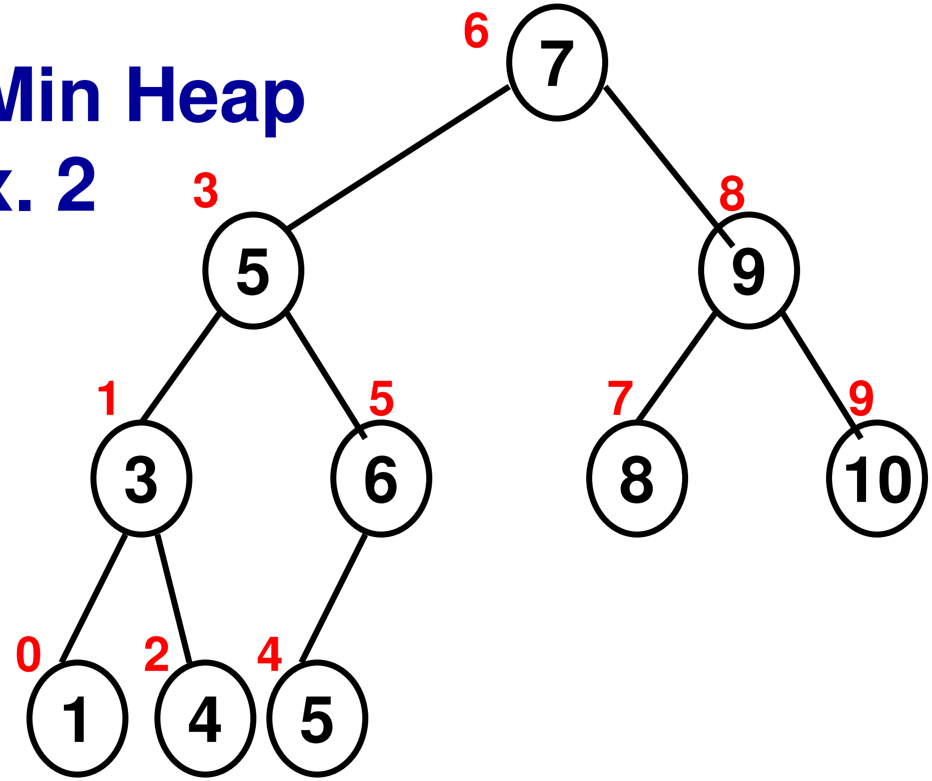
BST



BST to Min Heap Ex. 2

Given BST





Inorder-based node indices

0	1	2	3	4	5	6	7	8	9
1	3	4	5	5	6	7	8	9	10

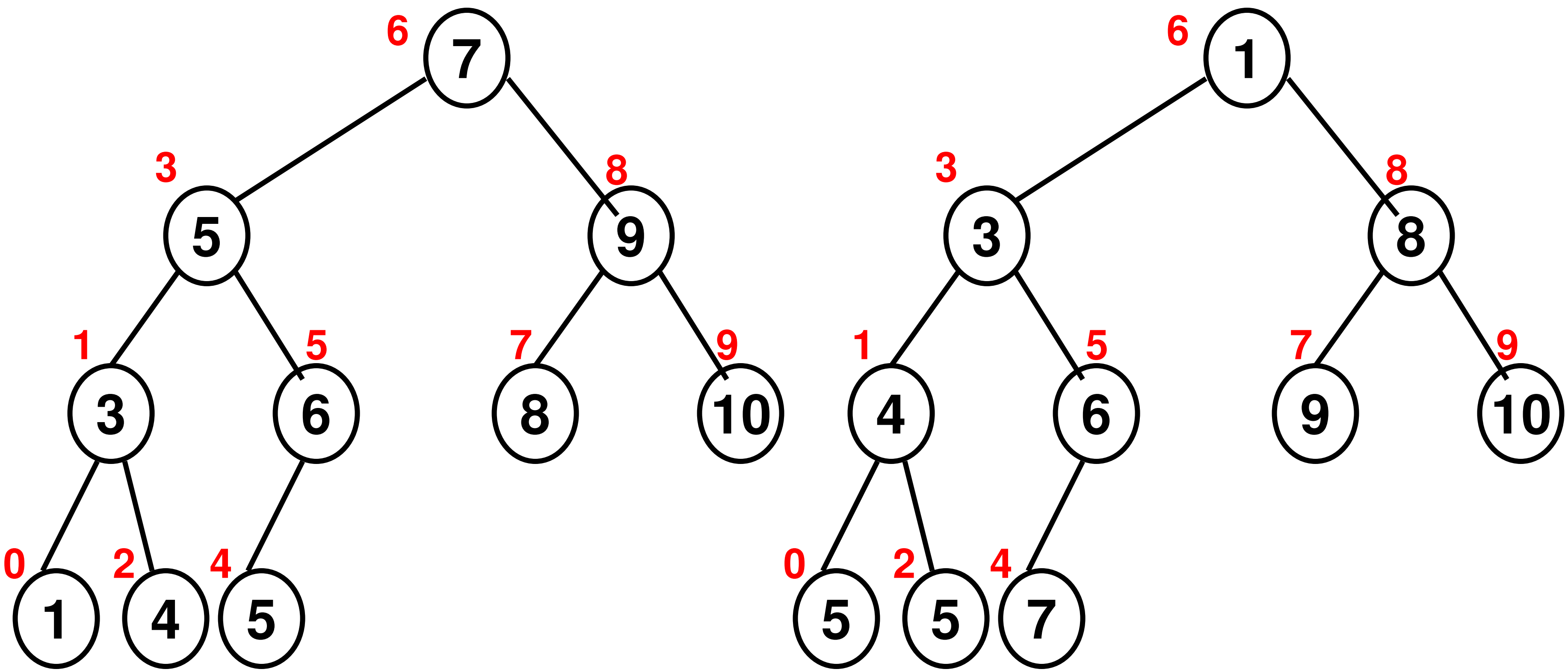
Preorder Listing of the node indices

6	3	1	0	2	5	4	8	7	9

Min Heap

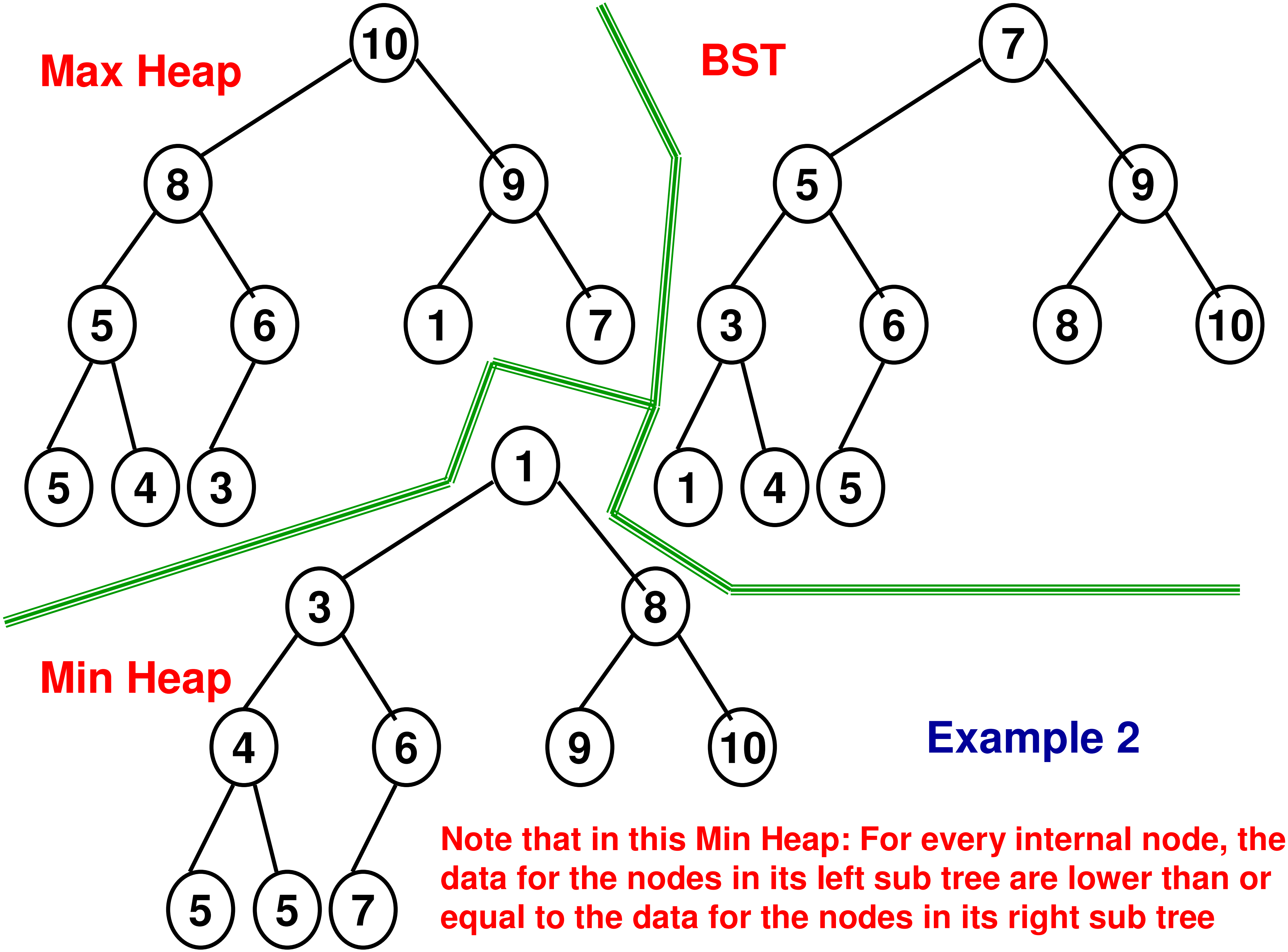
6	3	1	0	2	5	4	8	7	9
1	3	4	5	5	6	7	8	9	10

BST to Min Heap: Ex. 2



Min Heap

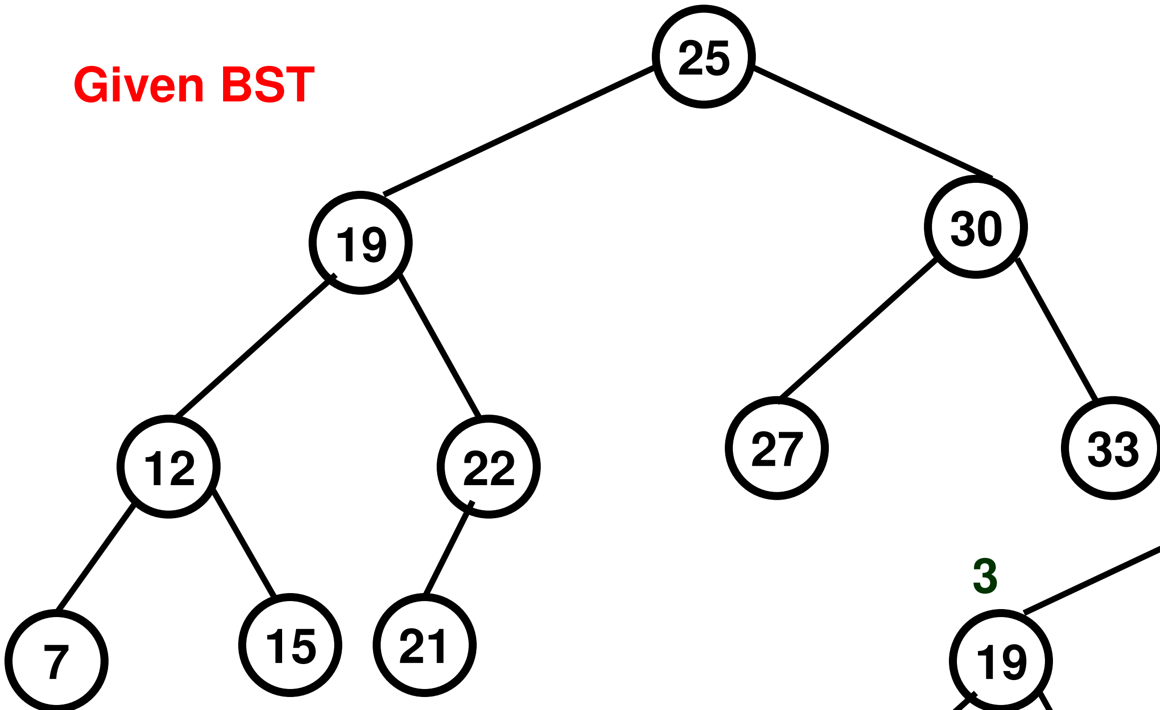
6	3	1	0	2	5	4	8	7	9
1	3	4	5	5	6	7	8	9	10



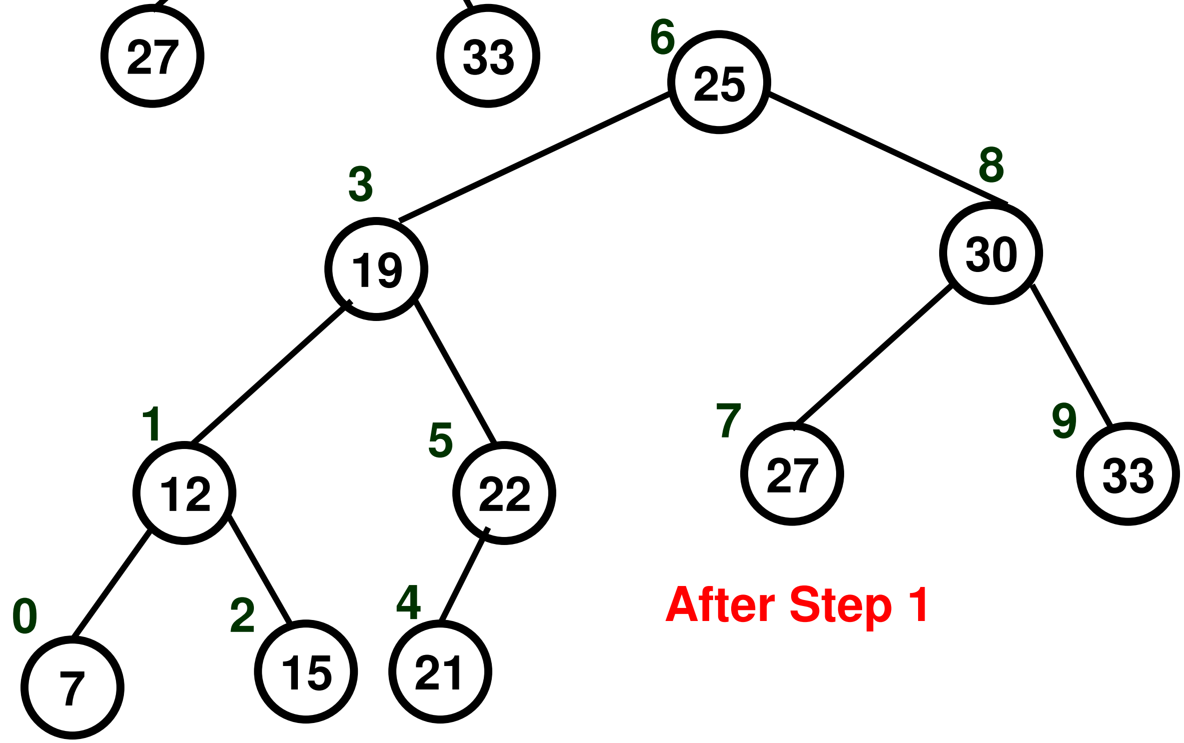
Note that in this Min Heap: For every internal node, the data for the nodes in its left sub tree are lower than or equal to the data for the nodes in its right sub tree

BST to Min Heap: Example 3

Given BST



Step 1: Do an inorder traversal of the BST and find the indexes of the nodes and the sorted order of the data



After Step 1

BST to Min Heap: Example 3

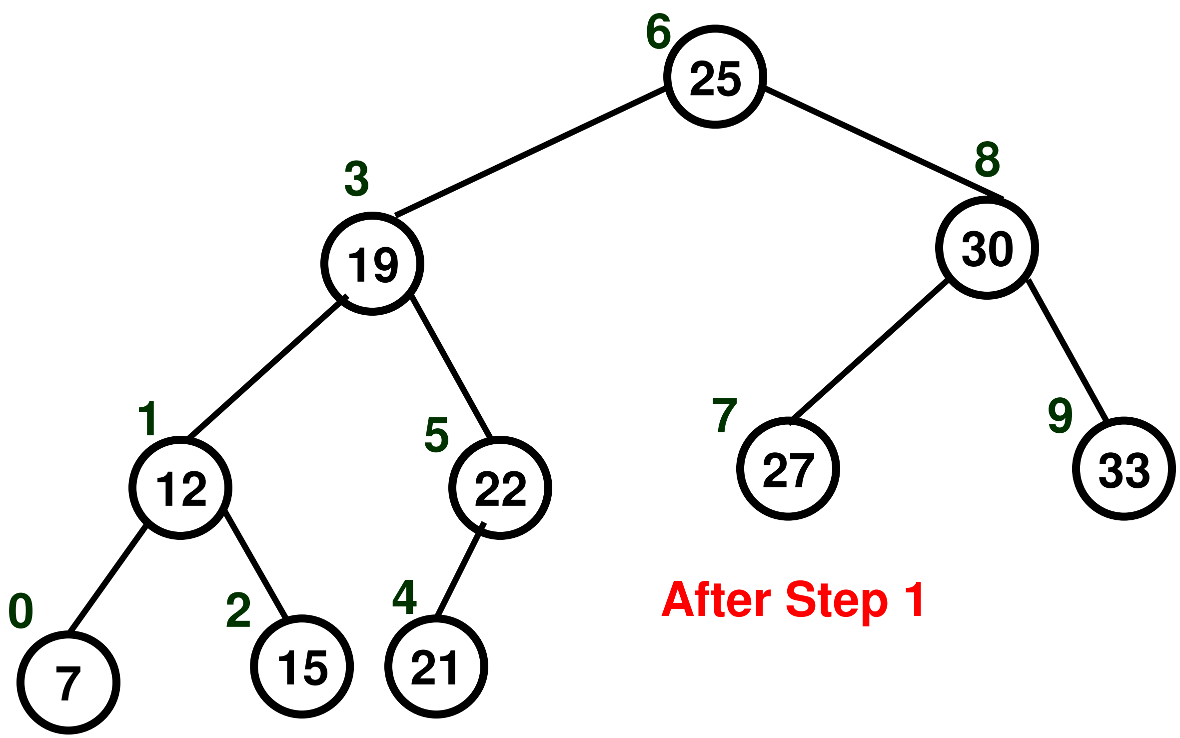
Sorted List of Integers (Inorder Array)

0	1	2	3	4	5	6	7	8	9
7	12	15	19	21	22	25	27	30	33

Preorder Array of Node Indices

6	3	1	0	2	5	4	8	7	9

Step 2: Do a preorder of the BST based on the identified node indices and replace the data at each node visited (as part of the preorder traversal) with the data from the inorder array



After Step 1

BST to Min Heap: Example 3

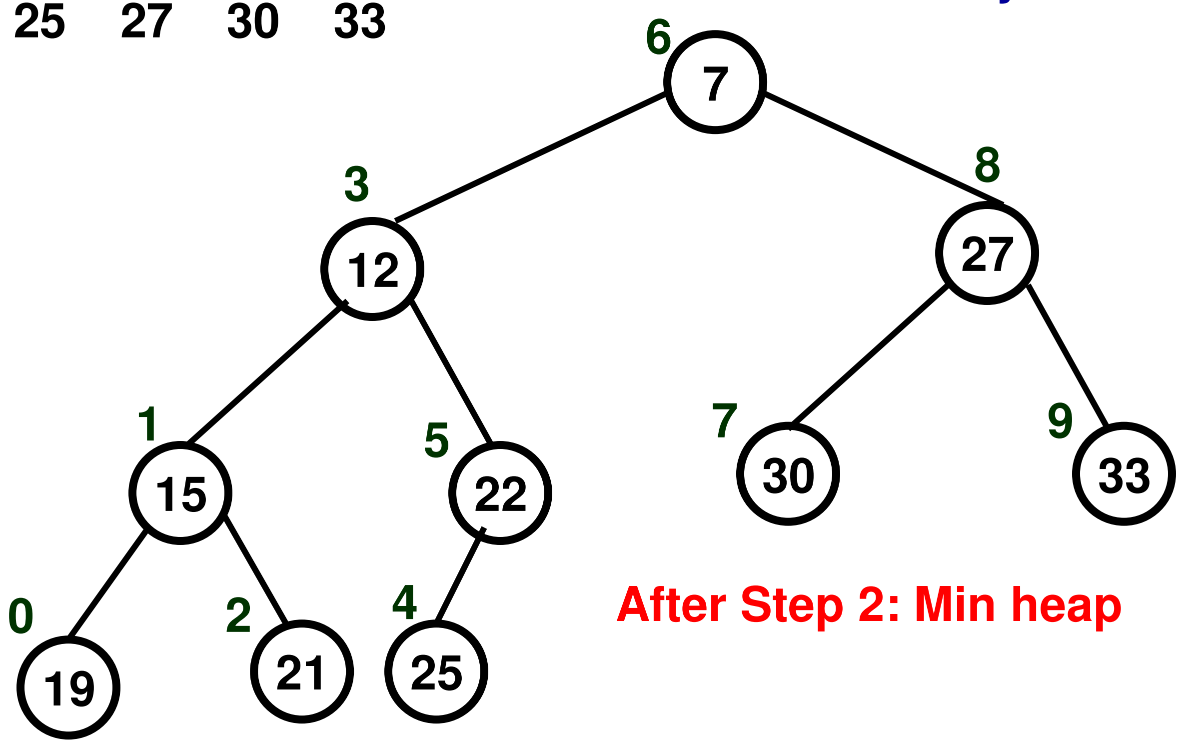
Sorted List of Integers (Inorder Array)

0	1	2	3	4	5	6	7	8	9
7	12	15	19	21	22	25	27	30	33

Preorder Array of Node Indices

6	3	1	0	2	5	4	8	7	9
7	12	15	19	21	22	25	27	30	33

Step 2: Do a preorder of the BST based on the identified node indices and replace the data at each node visited (as part of the preorder traversal) with the data from the inorder array



After Step 2: Min heap