

Q2-20 pts) For the given binary tree:

(a) Find the depth of all the leaf nodes and the height of all the internal nodes of the tree.

(b) Determine whether the binary tree is height-balanced or not. Show all the work.

(c) If the nodes are to be rearranged so that the height of the binary tree is the minimum, what is the minimum possible height of this binary tree of 10 nodes?

Level #	Node	Leaf node	depth	Internal node	height
0	(4)				
1	(3) 1, 2 (1)	3, 7	2	0, 1	4, 3
2	3 (0), (2) 4, 7 (0), 5 (0)	5, 9	2	2	1
3	(1) 6, 9 (0)	8	3	4	2
4	8 (0)		4	6	1

Height-balanced analysis

Internal node	Height of Left subtree	Height of Right subtree	Abs. Diff. in heights	Height-balanced
0	3	1	2	NO
1	0	2	2	NO
2	0	0	0	Yes
4	1	0	1	Yes
6	-1	0	1	Yes

$> 1 \Rightarrow \text{NO}$   
 $\leq 1 \Rightarrow \text{YES}$   
 The tree is NOT height balanced.

(c)  $H_{\min} = \lceil \log_2(N+1) \rceil - 1 = \lceil \log_2(11) \rceil - 1 = 3$

**CSC 228 Data Structures and Algorithms, Fall 2017**

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Quiz 4 (Oct. 16, 2017)

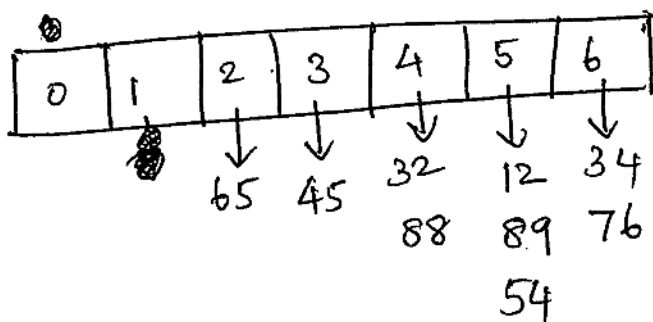
Max. Points: 30

Max. Time: 20 min.

Q1-10 pts) Consider an array  $A = \{45, 12, 34, 89, 76, 65, 54, 32, 88\}$  and hash function  $H(v) = v \text{ mod } 7$ .

- (a) Construct the hash table and show it.
- (b) Determine the average number of comparisons for a successful search
- (c) What is the maximum number of comparisons for an unsuccessful search?

$$H(v) = v \text{ mod } 7$$



Successful  
search  
# Comp.

- 1
- 2
- 3

Avg. # Comparisons for successful search = 
$$\frac{(5 \text{ keys})(1 \text{ comp}) + (3 \text{ keys})(2 \text{ comp}) + (1 \text{ key})(3 \text{ comp})}{9 \text{ keys}}$$

$$= \frac{5 + 6 + 3}{9} = \frac{14}{9} = 1.55$$

Max. # Comparisons for unsuccessful search = 3