Module 3: Greedy Strategy

1) Consider the greedy strategy applied for the "Activities Selection problem". Prove the following: At least one maximal conflict-free schedule includes the activity that finishes first.

Module 5: Graph Theory

1) Prove the following property with respect to shortest path trees and the Dijkstra algorithm:
When a vertex v is picked for relaxation, we have optimized the vertex (i.e., found the shortest path for the vertex from a source vertex s).

2) Prove the following necessary and sufficient conditions for the topological sort of a directed graph:
Necessary condition: The directed graph needs to be a DAG in order to generate a topological sort of the vertices
Sufficient condition: It is sufficient for a directed graph to be a DAG in order to generate a topological sort of the vertices

3) Prove that there exists only one minimum spanning tree for a graph with unique edge weights.

4) Prove that the Kruskal's algorithm when applied to determine a maximum spanning tree does find one.

Module 6: P, NP, NP-complete problems:

1) Define the following classes of problems. Draw a figure that comprehensively illustrates the relationship between these classes of problems.
P, NP, NP-hard and NP-complete.

2) Prove that the Hamiltonian circuit problem is polynomial-time reducible to the Traveling salesman problem.

3) Prove that the approximation ratio of the Twice-around-the-tree heuristic for the Traveling Salesman problem (TSP) is 2.0.

4) Given a problem, write down which of the following classes it belongs to?

Classes: P, NP (but not NP-hard), NP-hard (but not NP), NP-complete

Problems:
Sorting problem
Decision version of Traveling salesman problem (TSP)
Optimization version TSP
Decision version of minimum spanning tree problem (MST)
Optimization version of MST problem
Halting problem
Hamiltonian circuit problem
2-colorability problem
Shortest path problem
Maximum clique problem