Student Name: ______________________________ J#: ________________

CSC 323 Algorithm Design and Analysis, Fall 2018
Instructor: Dr. Natarajan Meghanathan

Quiz 3 (Take Home) **Total: 50 points**

Due: October 9, 2018 (11.30 AM, in-class). Quiz solutions submitted after 11.30 AM will not be accepted. Submit a printed hardcopy in class (with this quiz sheet as a cover page and your name and J# on the top of the sheet).

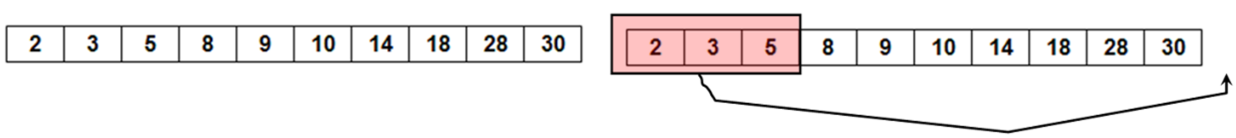
Note: Strictly, there should NOT be any copying. If the instructor finds that two or more quiz solutions involve some sort of copying, all the concerned students found to be involved in copying will get a zero.

Q1: 20 points) In this quiz, we will define a unimodal array as an array of 'distinct' integers wherein the array is a sequence of monotonically decreasing integers followed by a sequence of monotonically increasing integers. Design a Θ(logn) algorithm to determine the **minimum element** in the unimodal array.

(a) Show the pseudo code of your algorithm.
(b) Justify the correctness of the algorithm.
(c) Analyze the run-time complexity of the algorithm and show that it is Θ(logn).
(d) Show the working of the algorithm (along with the appropriate index values) for three different cases (in each case, the array should be of size at least 10 integers):

- i) the array is a sequence of monotonically decreasing integers followed by a sequence of monotonically increasing integers
- ii) the array is strictly a sequence of monotonically decreasing integers
- iii) the array is strictly a sequence of monotonically increasing integers

Q2: 30 points) Consider a sorted, but rotated array of distinct integers (i.e., no two integers are the same). For example, the following sorted array



when rotated three elements to the "left" becomes:

0	1	2	3	4	5	6	7	8	9
8	9	10	14	18	28	30	2	3	5

All the elements (except an element called the pivot at index p) of the sorted, but rotated array of integers have a property that they are less than the element to the right of them. Only the pivot element is greater than the element to the right of it. In the above sorted, but rotated array of integers, the pivot is the integer 30 at index 6. Incidentally, the pivot also happens to be the largest element in the array and is the last element in the original sorted array (before the rotation). In the above example, the pivot element 30 is the largest element of the array and is also the last element of the original sorted array (before the rotation).

(a) Design a binary search-based algorithm to identify the pivot in a sorted, but rotated array of integers.
(b) Extend the algorithm of (a) to do a successful search for a key that is present in the sorted, but rotated array.
(c) Extend the algorithm of (a) to do a unsuccessful search for a key that is not present in the sorted, but rotated array.
(d) For each of the algorithms in (a), (b) and (c), illustrate the execution of the algorithm for the array given in the problem statement.
(e) Analyze the time complexity of the algorithms of (a), (b) and (c).

Note that in addition to describing the working of your algorithms, you should also write the pseudo code for your algorithms of (a), (b) and (c).