Module 5: Graph Theory

- 1) Prove the following property with respect to shortest path trees and the Dijkstra algorithm: When a vertex v is picked for relaxation, we have optimized the vertex (i.e., found the shortest path for the vertex from a source vertex s).
- 2) Prove the following property associated with the working style of the Dijkstra algorithm: The weights of the vertices that are optimized in the Dijkstra algorithm are in the non-decreasing order.
- 3) Prove that there exists only one minimum spanning tree for a graph with unique edge weights.
- 4) Prove that the Kruskal's algorithm when applied to determine a maximum spanning tree does find one.

Module 6: P, NP, NP-complete problems:

- 1) Define the following classes of problems. Draw a figure that comprehensively illustrates the relationship between these classes of problems.
- P, NP, NP-hard and NP-complete.
- 2) Prove that the Hamiltonian circuit problem is polynomial-time reducible to the Traveling salesman problem.
- 3) Prove that the approximation ratio of the Twice-around-the-tree heuristic for the Traveling Salesman problem (TSP) is 2.0.
- 4) Given a problem, write down which of the following classes it belongs to?

Classes: P, NP (but not NP-hard), NP-hard (but not NP), NP-complete

Problems:

Maximum clique problem

Sorting problem
Decision version of Traveling salesman problem (TSP)
Optimization version TSP
Decision version of minimum spanning tree problem (MST)
Optimization version of MST problem
Halting problem
Hamiltonian circuit problem
2-colorability problem
Shortest path problem