$\qquad$

CSC 641 Network Science, Fall 2019
Instructor: Dr. Natarajan Meghanathan
Exam 5 (Take Home)
Due: Submit Hardcopy in class on Thursday, Nov. 21st @ 7.30 PM
Max. Points: 100
Answer all questions in the space provided.
Use additional sheets, if needed. Staple all the sheets together.

1) ( 6 pts ) Consider a regular ring lattice of degree 12 for every node. This regular graph is transformed to a small-world network by arbitrarily re-wiring the edges with probability $\beta$. Let the clustering coefficient of the small-world network generated out of this re-wiring be 0.3 . Determine the re-wiring probability $\beta$.

Student Name:
J\#: $\qquad$
2) ( 12 pts ) At time 300 units, you are given the degree distribution of the nodes that joined at the time units indicated as shown. Determine the number of links added per node introduction (m) and the network's dynamical exponent ( $\beta$ ). Also, estimate the degree of a node that joined the network at time 40 units.

| Node joining <br> Time, $\boldsymbol{t}_{\boldsymbol{i}}$ | Degree at time <br> $\mathbf{t}=\mathbf{3 0 0}$ |
| :--- | :--- |
| 5 | 29 |
| 15 | 16 |
| 30 | 11 |
| 45 | 9 |
| 60 | 7 |
| 75 | 6 |
| 100 | 5 |

$\qquad$
3) ( 13 pts ) A node joined the network at time 20 units. You are given the degree of the node at various time units. Determine the number of links added per node introduction and the fitness $(\eta)$ of the node. Under the BB model of evolution, assume the dynamical exponent value for a node is equal to the fitness of the node itself. Also, estimate the degree of the node at time 300 units.

Time Unit, $t$ Degree at Time $t$ 30 8 $60 \quad 29$
$90 \quad 60$
120
$150 \quad 150$
180
209
$200 \quad 252$
225
312
$\qquad$
4) ( 12 pts ) Consider the BB model for evolution of scale-free networks.

- Let the parameter $\beta(\eta i)$ for any node $i$ be equal to the fitness of node $i, \eta i$. Consider two nodes A and $B$ such that the fitness of node B is four times the fitness of node A.
- Node A joins the network at time 20 units and node B joins the network at time 150 units.
- If the degree of the nodes increase for every time unit (when a new node joins), what is the minimum value of the time unit starting from which the degree of node B would always be greater than the degree of node A?

Student Name: $\qquad$ J\#: $\qquad$
5) (7 pts) Consider a network modeled using the power-law, $\mathrm{P}(\mathrm{K})=\mathrm{CK}^{-\gamma}$. Determine the power-law exponent $\gamma$ and the constant C if the network has approximately $15 \%$ of nodes with degree 4 and $5 \%$ of nodes with degree 6.

Student Name: $\qquad$ J\#: $\qquad$
6) (10 pts) Given the following adjacency list for the vertices, Use the Kurtosis measure to determine whether the Degree distribution could be classified to exhibit "scale-free" property.

| 0 | 1 |
| :--- | :--- |
| 0 | 2 |
| 0 | 3 |
| 0 | 4 |
| 0 | 5 |
| 0 | 6 |
| 0 | 7 |
| 1 | 2 |
| 1 | 3 |
| 1 | 4 |
| 1 | 5 |
| 1 | 8 |
| 1 | 9 |$\quad$| 2 | 3 |
| :--- | :--- |
| 2 | 8 |
| 3 | 4 |
| 3 | 5 |
| 3 | 6 |
| 3 | 9 |
| 4 | 6 |
| 4 | 7 |
| 4 | 8 |
| 6 | 7 |
| 6 | 9 |

Student Name: $\qquad$ J\#: $\qquad$
7) (12 pts) Consider the following degree distribution of the nodes and their fitness.
(a) Determine the probability with which each node is likely to get the first link with a newly joining node under the BB model.
(b) Let a new node join the network with 2 links under the BB model. Determine which nodes are likely to get connected to the new node.

| ID | Degree | Fitness |
| :--- | :--- | :--- |
| 1 | 5 | 2 |
| 2 | 2 | 4 |
| 3 | 5 | 8 |
| 4 | 3 | 5 |
| 5 | 3 | 3 |
| 6 | 2 | 5 |
| 7 | 2 | 4 |

$\qquad$ J\#: $\qquad$
8) (10 pts) Let the network (shown on the right side) evolve to a small-world network under the "Enhanced Watts-Strogatz Model". Let the link 1-2 be the first link chosen for rewiring. Predict the vertex to which vertex 1 will be rewired to? Use $\mathrm{q}=1$


Student Name: $\qquad$ J\#: $\qquad$
9) ( 6 pts ) Consider the enhanced WS model for small-world networks. Let there be a regular graph that is transformed to a small-world network. For every edge $(u, v)$ selected for re-wiring, the probability that a node $w$ of distance 3 hops to $u$ is picked for re-wiring is 0.25 and the probability that a node $w^{\prime}$ of distance 5 hops to $u$ is picked for re-wiring is 0.10 . Find the value for the parameter $\boldsymbol{q}$ in the enhanced WS model.
$\qquad$ J\#: $\qquad$
10) (12 pts) Given the probability degree distribution:
a) Draw a plot of the degree distribution and determine if the degree distribution follows a power-law or Poisson pattern?
b) Determine the parameters of the degree distribution you decided as listed below.
-- If the degree distribution follows Power Law, determine the Power Law
Constant (C) and the Power Law Exponent ( $\gamma$ )
-- If the degree distribution follows Poisson Law, determine the randomness
index, mean and standard deviation of node degree
$\mathrm{K} \quad \mathrm{P}(\mathrm{K})$
10.7780
20.1240
$3 \quad 0.0423$
$4 \quad 0.0197$
$5 \quad 0.0109$
$6 \quad 0.0067$
$7 \quad 0.0045$
$8 \quad 0.0031$

