# Module 2:
# List ADT

Dr. Natarajan Meghanathan
Professor of Computer Science
Jackson State University
Jackson, MS 39217
E-mail: natarajan.meghanathan@jsums.edu

# List ADT

- A collection of entities of the same data type
- List ADT (static)
  - Functionalities (logical view)
    - Store a given number of elements of a given data type
    - Write/modify an element at a particular position
    - Read an element at a particular position
- Implementation:
  - Arrays: A contiguous block of memory of a certain size, allocated at the time of creation/initialization
    - Time complexity to read and write/modify are $\Theta(1)$ each

A[0]　　　　A[2]

| Array index | 0 | 1 | 2 | 3 | ……. | N-1 |
|---|---|---|---|---|---|---|
| Array, A | 10 | 23 | 13 | 17 | ……. | 21 |
| Memory address | 200 | 204 | 208 | 212 | | 2xx |

# Code 1(C++): Static List Implementation using Arrays

```cpp
#include <iostream>
using namespace std;
```

```cpp
class List{
   private:
      int *array;

   public:
      List(int size){
              array = new int[size];
      }

      void write(int index, int data){
              array[index] = data;
      }

      int read(int index){
              return array[index];
      }

};
```

```cpp
int main(){

   int listSize;

   cout << "Enter list size: ";
   cin >> listSize;

   List integerList(listSize);

   for (int i = 0; i < listSize; i++){
           int value;
           cout << "Enter element # " << i << " : ";
           cin >> value;
           integerList.write(i, value);
   }
return 0;
}
```

# Dynamic List ADT

- **Limitations with Static List**
  - The list size is fixed (during initialization); cannot be increased or decreased.
  - A new element cannot be inserted (if the list is already full) or an existing element cannot be deleted.
- **Key Features of a Dynamic List**
  - Be able to resize (increase or decrease) the list at run time. The list size need not be decided at the time of initialization. We could start with a list of size one and populate it as elements are to be added.
  - Be able to insert or delete an element at a particular index at any time.
- **Performance Bottleneck**
  - When we increase the size of the list (i.e., increase the size of the array that stores the elements), the contents of the array need to be copied to a new memory block, element by element. ➔ $O(n)$ time.
  - Hence, even though, we could increase the array size by one element at a time, the 'copy' operation is a performance bottleneck and the standard procedure is to double the size of the array (list) whenever the list gets full.
  - A delete operation also takes $O(n)$ time as elements are to be shifted one cell to the left.

# Code 2: Code for Dynamic List ADT Implementation using Arrays

**Variables and Constructor (C++)**

```cpp
private:
  int *array;
  int maxSize;
  int endOfArray;

public:
  List(int size){
        maxSize = size;
        array = new int[maxSize];
        endOfArray = -1;
  }
```

**Function to free the memory (C++)**

```cpp
void deleteList(){
    delete[] array;
}
```

**isEmpty (C++)**

```cpp
bool isEmpty(){

    if (endOfArray == -1)
            return true;

    return false;
}
```

# Code 2: Insert Function (C++)

```
void insertAtIndex(int insertIndex, int data){

        // if the user enters an invalid insertIndex, the element is
        // appended to the array, after the current last element
        if (insertIndex > endOfArray+1)
                insertIndex = endOfArray+1;

        if (endOfArray == maxSize-1)
                resize(2*maxSize);

        for (int index = endOfArray; index >= insertIndex; index--)
                array[index+1] = array[index];

        array[insertIndex] = data;
        endOfArray++;

}
```

**Will take O(n) time each, where n = maxSize + 1**

```
void insert(int data){
        if (endOfArray == maxSize-1)
                resize(2*maxSize);

        array[++endOfArray] = data;

}
```

# Code 2: Resize Function (C++)

```cpp
void resize(int s){

        // in addition to increasing, the resize function
        // also provides the flexibility to reduce the size
        // of the array


        int *tempArray = array;

        array = new int[s];

        for (int index = 0; index < min(s, endOfArray+1); index++){
                array[index] = tempArray[index];
        }


        maxSize = s;
}
```

**Have another pointer (a temporary ptr) to refer to the starting address of the memory represented by the original array**

**Allocating a new set of memory blocks to the 'array' variable**

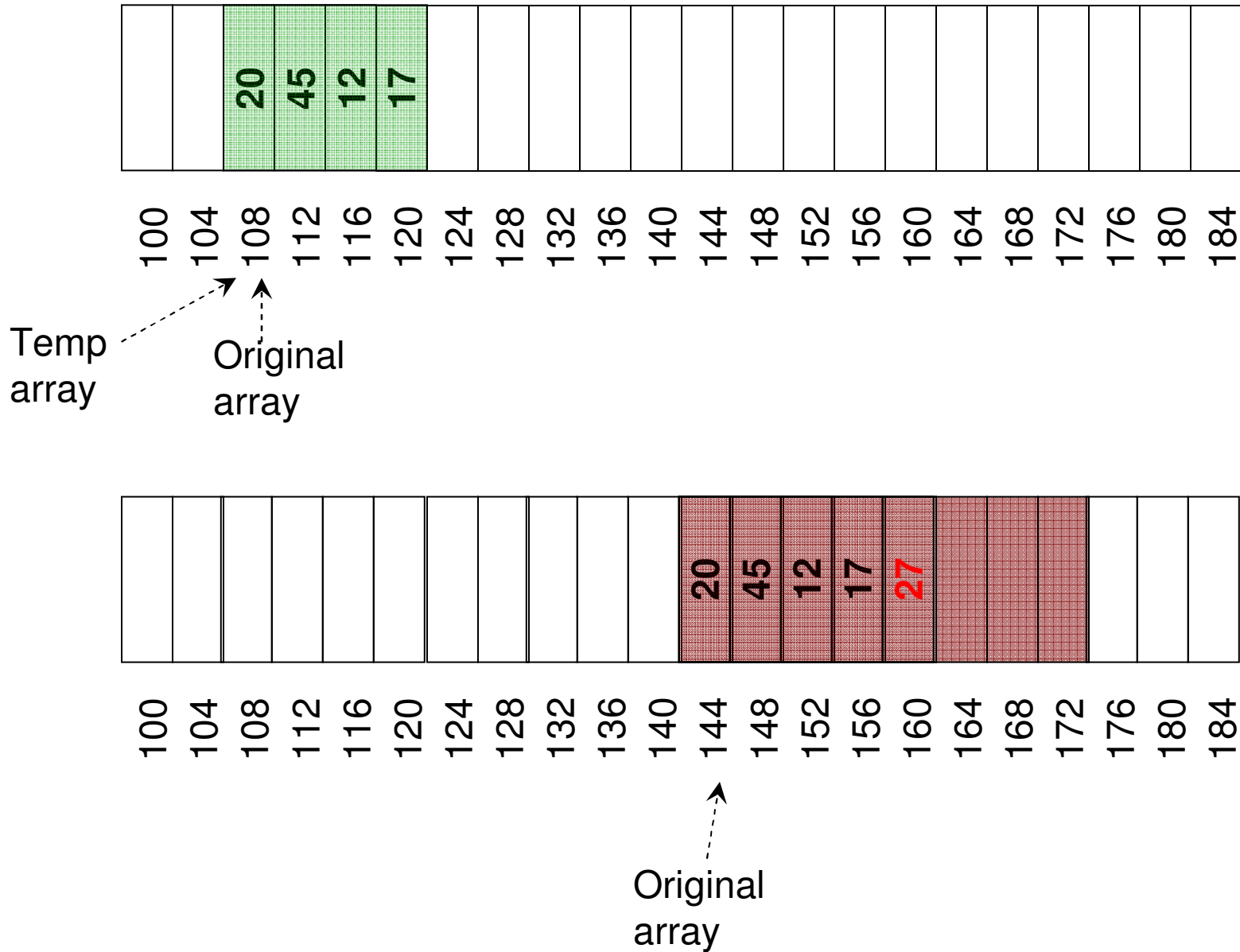**Copying back the contents pointed to by the temporary array pointer to the original array**

**If the array size is reduced from maxSize to s, only the first 's' elements are copied. Otherwise, all the maxElements are copied**

**new value of maxSize**

**Note: Include <algorithm> header file if the min function is not automatically loaded to your computing environment.**

# Insert Operation
## (incl. Relocation and Doubling the Size of the Array)

| | | 20 | 45 | 12 | 17 | | | | | | | | | | | | | | | | |

100 104 108 112 116 120 124 128 132 136 140 144 148 152 156 160 164 168 172 176 180 184

Temp
array

Original
array

| | | | | | | | | | | 20 | 45 | 12 | 17 | 27 | | | | | | | |

100 104 108 112 116 120 124 128 132 136 140 144 148 152 156 160 164 168 172 176 180 184

Original
array

# Time complexity analysis for 'Insert': Dynamic List ADT as an Array

**Insert operation**

**(i) Worst case:** If the element is to be inserted as the first element in the array, then elements from index endOfArray(eoA) to index '0' have to be shifted one position to the right. If eoA = n-1, then 'n' (indexes 0 to n-1) such shifting need to be done.
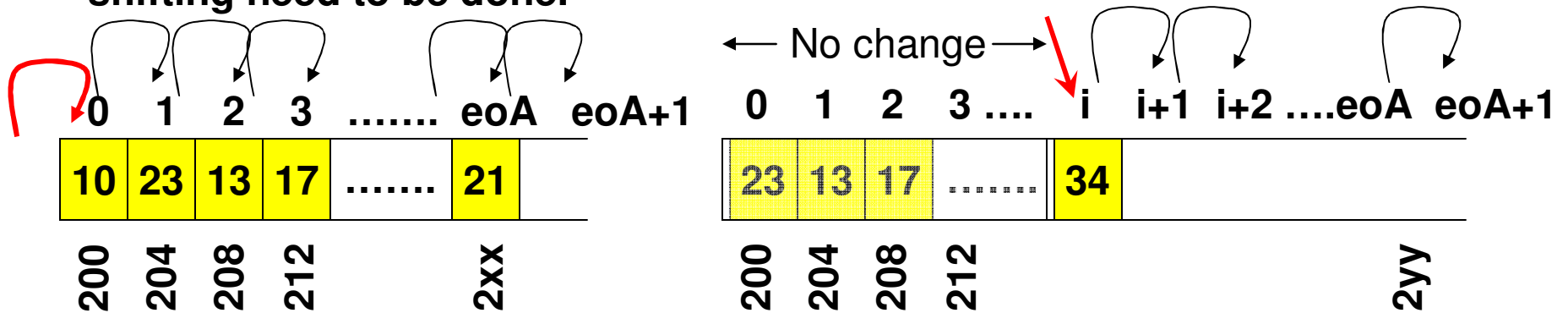


**(ii) Best case:** If the element is to be inserted at the end of the array, no shifting is needed.

**(iii) In general,** if the element is to be inserted at index i, then the elements from index endOfArray(eoA) to index 'i' need to be shifted one cell to the right.

**Time complexity for insert operation: O(n)**

# Code 2: Other Auxiliary Functions

**(C++)**

```cpp
int read(int index){
        return array[index];
}

void modifyElement(int index, int data){
        array[index] = data;
}

void deleteElement(int deleteIndex){
        // shift elements one cell to the left starting from
        // deleteIndex+1 to endOfArray-1
        // i.e., move element at deleteIndex + 1 to deleteIndex and so on

        for (int index = deleteIndex; index < endOfArray; index++)
                array[index] = array[index+1];

        endOfArray--;
}

int countList(){
        int count = 0;
        for (int index = 0; index <= endOfArray; index++)
                count++;

        return count;
}
```

# Time complexity analysis for 'Delete': Dynamic List ADT as an Array

**Delete operation**

(i) **Worst case:** If the element to be deleted is the first element (at index 0) in the array, then the subsequent elements have to be shifted one position to the left, starting from index 1 to endOfArray (eoA) . If eoA = n-1, then n-1 such shifting need to be done.
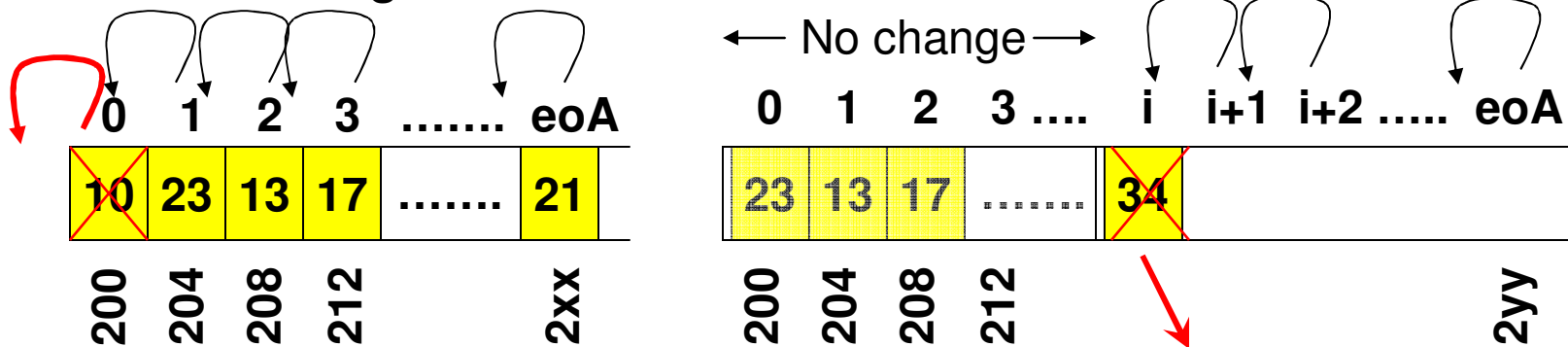


(ii) **Best case:** If the element to be deleted is at the end of the array, no shifting is needed.

(iii) **In general, if the element to be deleted is at index i, then the elements from index i+1 to endOfArray need to be shifted one cell to the left.**

## Time complexity for delete operation: O(n)

# Code 2: C++ main function

```cpp
int main(){

    int listSize;

    cout << "Enter list size: ";
    cin >> listSize;

    List integerList(1);

    for (int i = 0; i < listSize; i++){

        int value;
        cout << "Enter element # " << i << " : ";
        cin >> value;

        integerList.insert(i, value);
    }
}
```

**We will set the maximum size of the list to 1 and double it as and when needed**

# Pros and Cons of Implementing Dynamic List using Array

- Pros: $\Theta(1)$ time to read or modify an element at a particular index
- Cons: O(n) time to insert or delete an element (at any arbitrary position)

- Note: Array is a contiguous block of memory
- When we double the array size (to handle the need for more space), the memory management system of the OS needs to search for contiguous blocks of memory that is double the previous array size.
  - Sometimes, it becomes difficult to allocate a contiguous block of memory, if the requested array size is larger.
- After we double the size (say from 50,000 to 100,000 to insert just one more element), the rest of the array remains unused. However, increasing the size of the array one element at a time is time consuming too. [Space-time tradeoff]
  - The copy operation involved during resizing the array is also time consuming

## Code 6: Run Time Complexity Analysis

```cpp
#include <iostream>
#include <ctime>
#include <ratio>
#include <chrono>
using namespace std;

int main ()
{
  using namespace std::chrono;

  high_resolution_clock::time_point t1 = high_resolution_clock::now();

  cout << "printing out 1000 stars...\n";
  for (int i=0; i<1000; ++i) cout << "*";
  cout << endl;

  high_resolution_clock::time_point t2 = high_resolution_clock::now();

  duration<double, std::nano> time_span_nano = t2 - t1;
  duration<double, std::micro> time_span_micro = t2 - t1;
  duration<double, std::milli> time_span_milli = t2 - t1;

  cout << "It took me " << time_span_nano.count() << " nanoseconds." << endl;
  cout << "It took me " << time_span_micro.count() << " microseconds." << endl;
  cout << "It took me " << time_span_milli.count() << " milliseconds." << endl;
  cout << endl;

  return 0;
}
```

# Linked List

- A Linked List stores the elements of the 'List' in separate memory locations and we keep track of the memory locations as part of the information stored with an element (called a node).
  - A 'node' in a Linked List contains the data value as well as the address of the next node.
- Singly Linked List: Each node contains the address of the node with the subsequent value in the list. There is also a head node that points to the first node in the list.
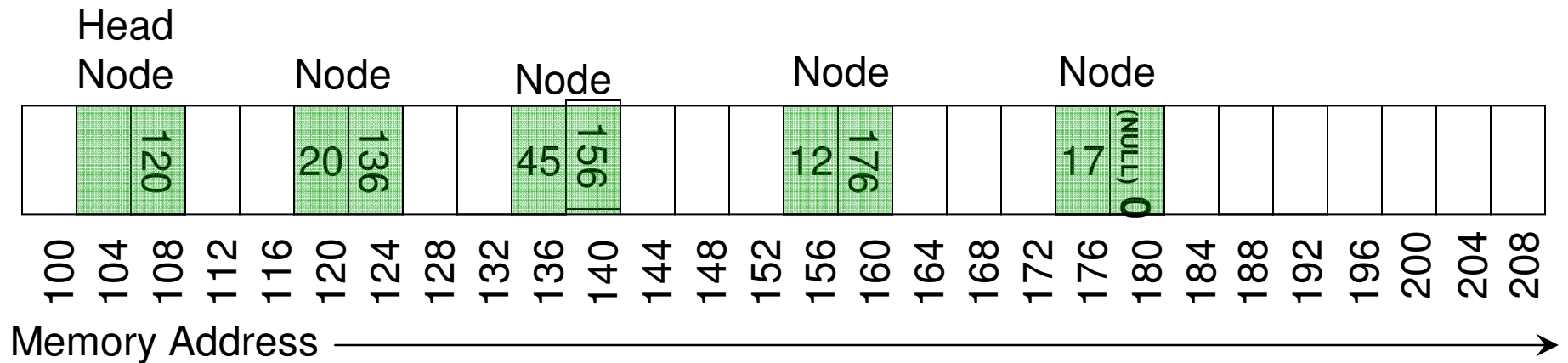
  Data      **With singly linked list – we can traverse only in one direction**
  nextNodePtr
- Doubly Linked List: Each node contains the address of the node with the subsequent value as well as the address of the node with the preceding value. There is also a head node pointing to the first node in the list and a tail node pointing to the last node in the list.
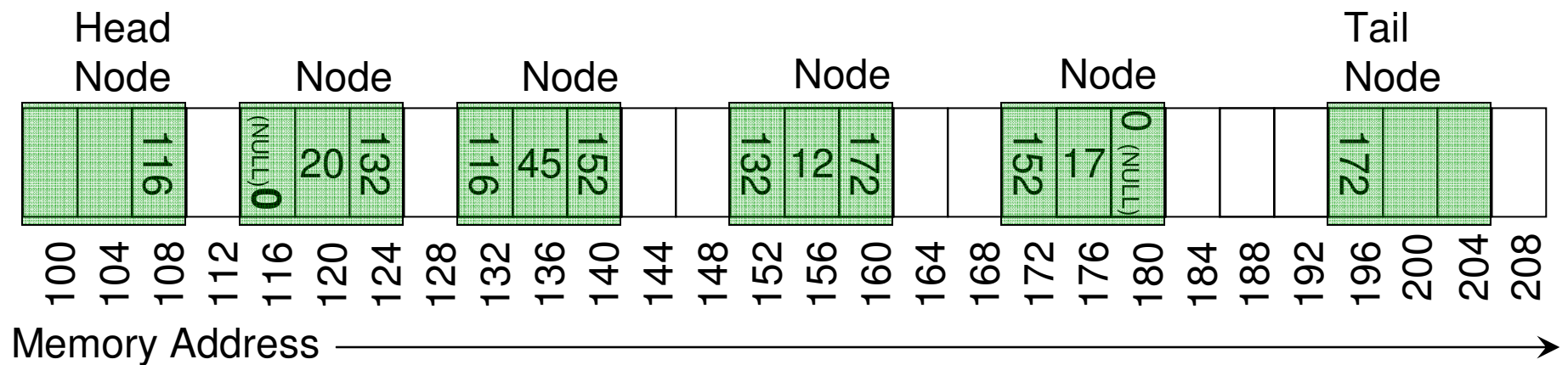
  prevNodePtr
  Data      **With doubly linked list – we can traverse in both directions**
  nextNodePtr

- Note: Memory address can be represented in 4 bytes. Hence, each pointer or reference to a memory will take 4 bytes of space.

# Singly Linked List



Head
Node | Node | Node | Node | Node

| 120 | 20 136 | 45 156 | 12 176 | 17 (NULL) 0 |

Memory Address →
100 104 108 112 116 120 124 128 132 136 140 144 148 152 156 160 164 168 172 176 180 184 188 192 196 200 204 208

# Doubly Linked List



Head
Node | Node | Node | Node | Node | Tail Node

| 116 | (NULL) 0 20 132 | 116 45 152 | 132 12 172 | 152 17 0 (NULL) | 172 |

Memory Address →
100 104 108 112 116 120 124 128 132 136 140 144 148 152 156 160 164 168 172 176 180 184 188 192 196 200 204 208

# Singly Linked List Implementation (Code 3)
## Class Node

**C++**

```cpp
private:
        int data;
        Node* nextNodePtr;
```

```cpp
public:
   Node(){}

   void setData(int d){
           data = d;
   }

   int getData(){
           return data;
   }
```

```cpp
public:
   void setNextNodePtr(Node* nodePtr){
                   nextNodePtr = nodePtr;
   }

   Node* getNextNodePtr(){
           return nextNodePtr;
   }
```
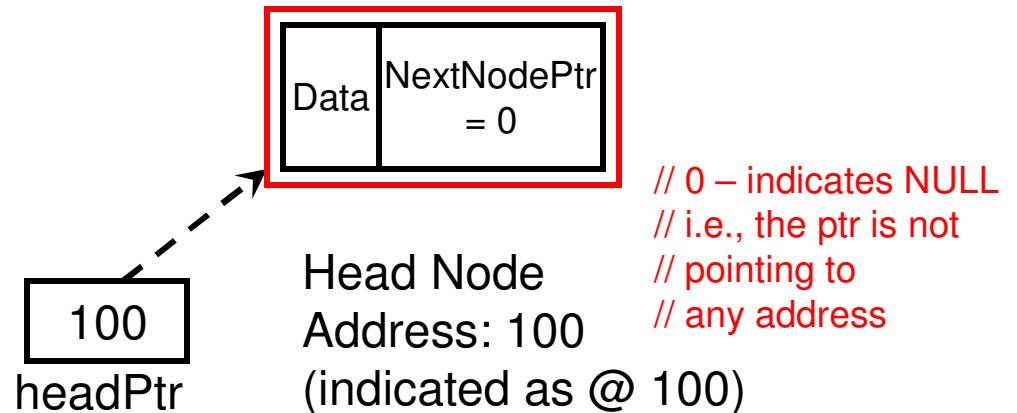
# Singly Linked List: Class List

## Class List (C++)
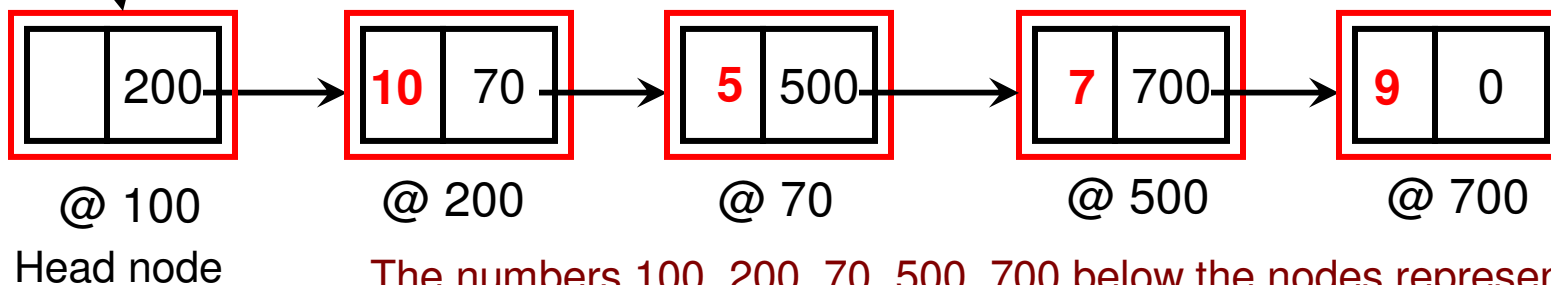
```
private:
    Node *headPtr;

public:        /* Note that the data for the
    List(){      Head node is not set */
        headPtr = new Node();
        headPtr->setNextNodePtr(0);
    }
```

## Initialization of List Object

| Data | NextNodePtr = 0 |
|------|-----------------|

// 0 – indicates NULL
// i.e., the ptr is not
// pointing to
// any address

```
100
```
headPtr

Head Node
Address: 100
(indicated as @ 100)

headPtr
100

Convention used to represent a Linked List.
Let the List be **10  5  7  9**

| | 200 | | 10 | 70 | | 5 | 500 | | 7 | 700 | | 9 | 0 |
|--|-----|--|----|----|--|---|-----|--|---|-----|--|---|---|

@ 100          @ 200          @ 70          @ 500          @ 700
Head node

The numbers 100, 200, 70, 500, 700 below the nodes represent
the address at which these nodes are stored, indicated with an @ symbol

## Class List (C++)

```cpp
void insert(int data){

    Node* currentNodePtr = headPtr->getNextNodePtr();
    Node* prevNodePtr = headPtr;

    while (currentNodePtr != 0){
        prevNodePtr = currentNodePtr;
        currentNodePtr = currentNodePtr->getNextNodePtr();
    }

    Node* newNodePtr = new Node();
    newNodePtr->setData(data);
    newNodePtr->setNextNodePtr(0);
    prevNodePtr->setNextNodePtr(newNodePtr);

}
```

Move the currentNode ptr from first node in the list to end of the list. When we come out of the 'while' loop, the prevNode ptr is the last node in the list and currentNode ptr points to null (0).

If the nextNodePtr for the headPtr points to null (0), then the list is empty. Otherwise, the list has at least one node.

```cpp
bool isEmpty(){

    if (headPtr->getNextNodePtr() == 0)
            return true;

    return false;

}
```
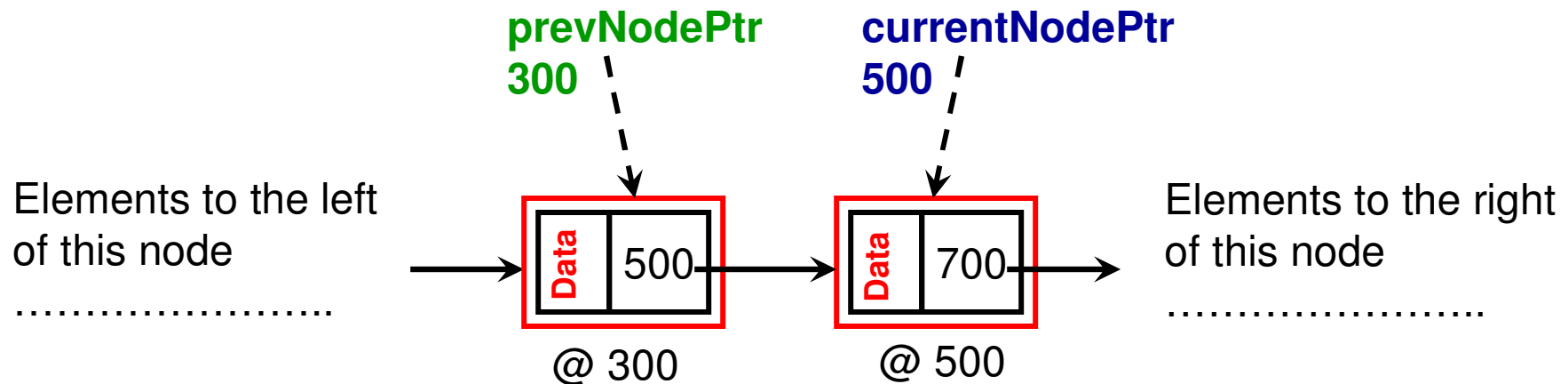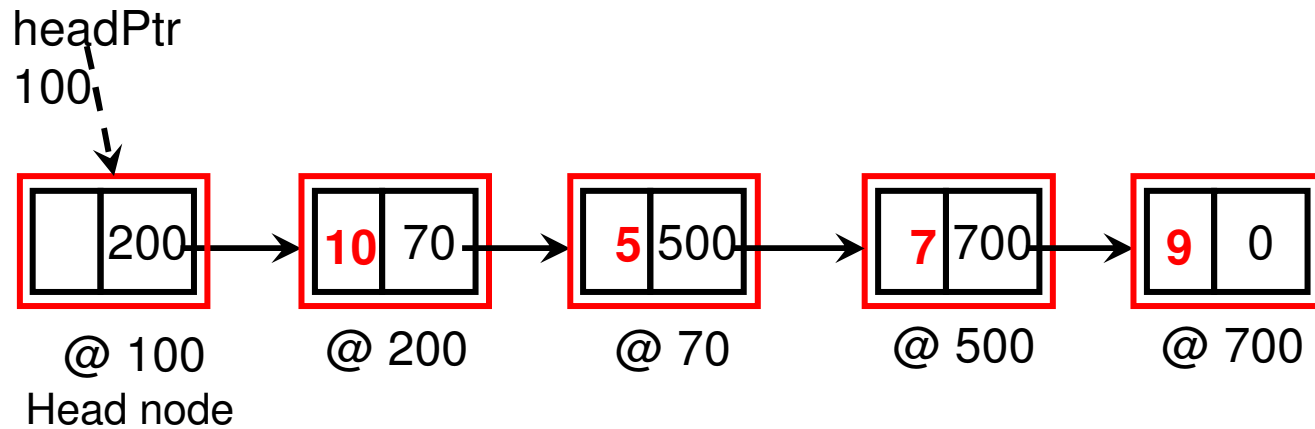
# prevNodePtr and currentNodePtr

- As we traverse through the list, node by node, we will maintain two pointers: the prevNodePtr and currentNodePtr.
  - The currentNodePtr has the address for the node that is currently being visited/ processed.
  - The prevNodePtr has the address for the node that was just visited before the current node.
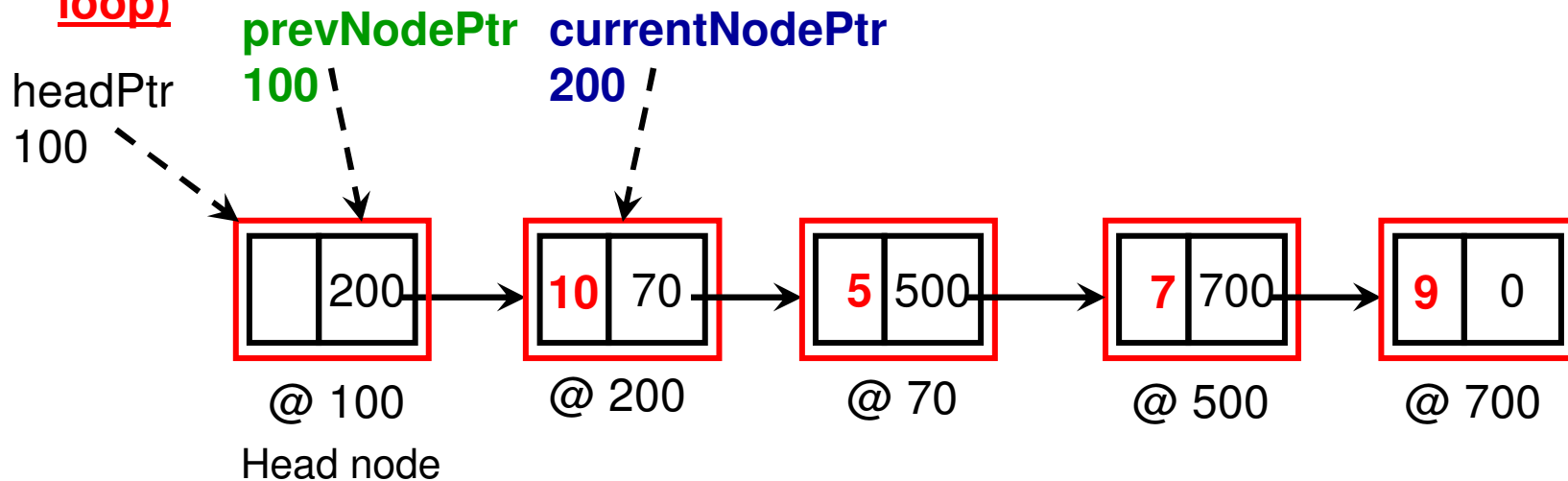- We have reached the end of the list when currentNodePtr is 0 (i.e., does not point to any node).

**prevNodePtr**
**300**

**currentNodePtr**
**500**

Elements to the left
of this node
…………………..

| Data | 500 |

@ 300

| Data | 700 |

@ 500

Elements to the right
of this node
………………..

# Example: Insertion at the End of the List (1)

Let the List be **10  5  7  9** and now we want to insert element '30' at the end.

headPtr
100

| | 200 | | → | **10** | 70 | → | **5** | 500 | → | **7** | 700 | → | **9** | 0 |

@ 100   @ 200   @ 70   @ 500   @ 700
Head node

**Initialization of prevNodePtr and currentNodePtr (before the while loop)**

```
Node* currentNodePtr = headPtr->getNextNodePtr();
Node* prevNodePtr = headPtr;
```

**prevNodePtr**   **currentNodePtr**
**100**             **200**

headPtr
100

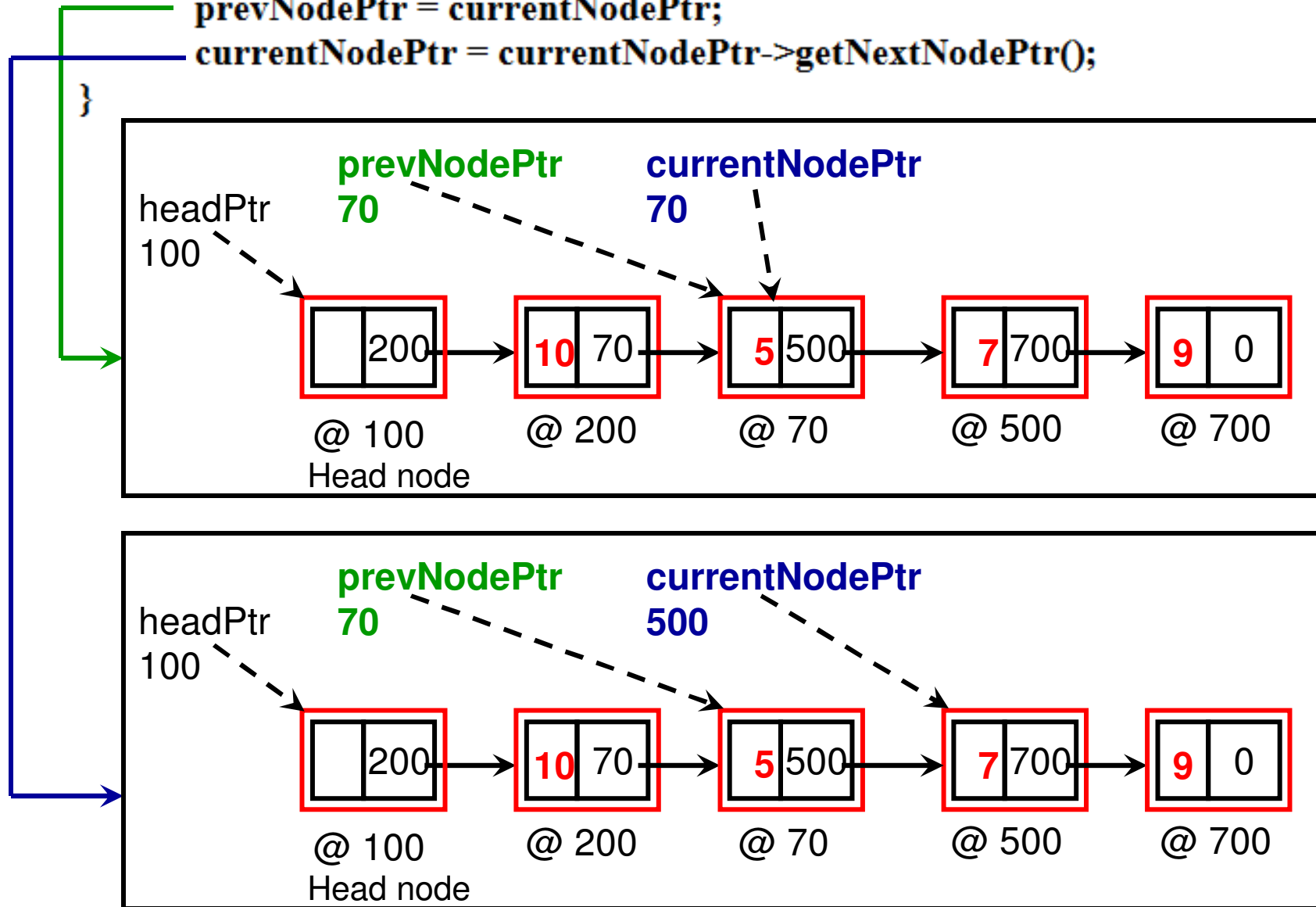| | 200 | | → | **10** | 70 | → | **5** | 500 | → | **7** | 700 | → | **9** | 0 |

@ 100   @ 200   @ 70   @ 500   @ 700
Head node

```
while (currentNodePtr != 0){
        prevNodePtr = currentNodePtr;
        currentNodePtr = currentNodePtr->getNextNodePtr();
}
```

**prevNodePtr**    **currentNodePtr**
**200**      **200**

headPtr
100

| | 200 | → | 10 | 70 | → | 5 | 500 | → | 7 | 700 | → | 9 | 0 |

@ 100     @ 200     @ 70     @ 500     @ 700
Head node

**prevNodePtr**    **currentNodePtr**
**200**          **70**

headPtr
100

| | 200 | → | 10 | 70 | → | 5 | 500 | → | 7 | 700 | → | 9 | 0 |

@ 100     @ 200     @ 70     @ 500     @ 700
Head node
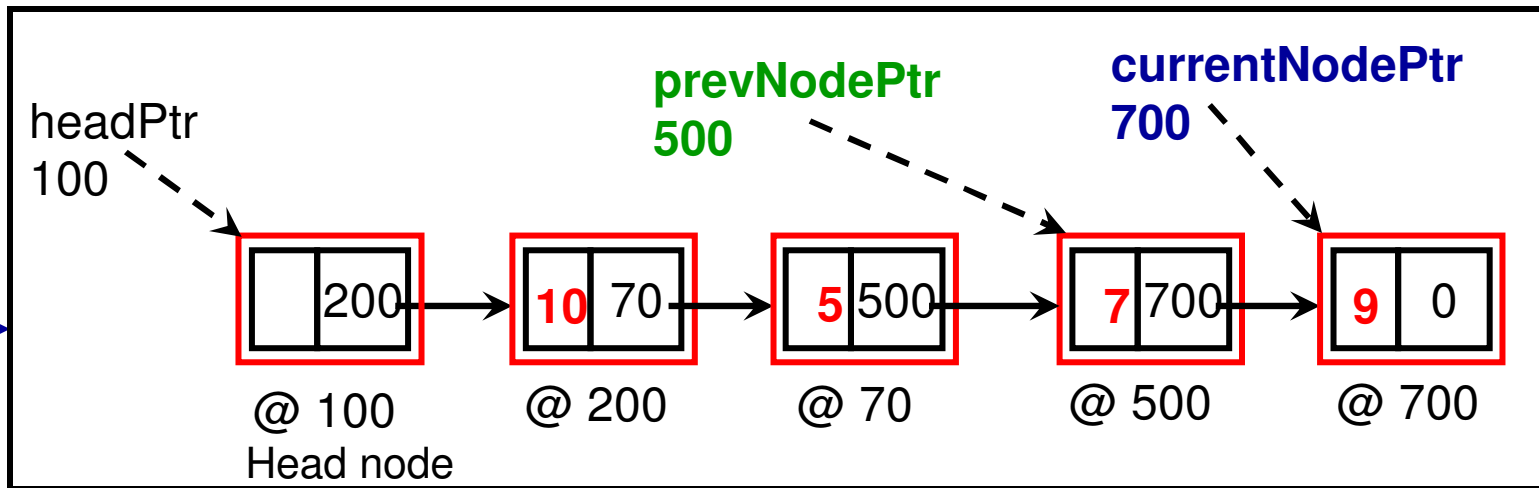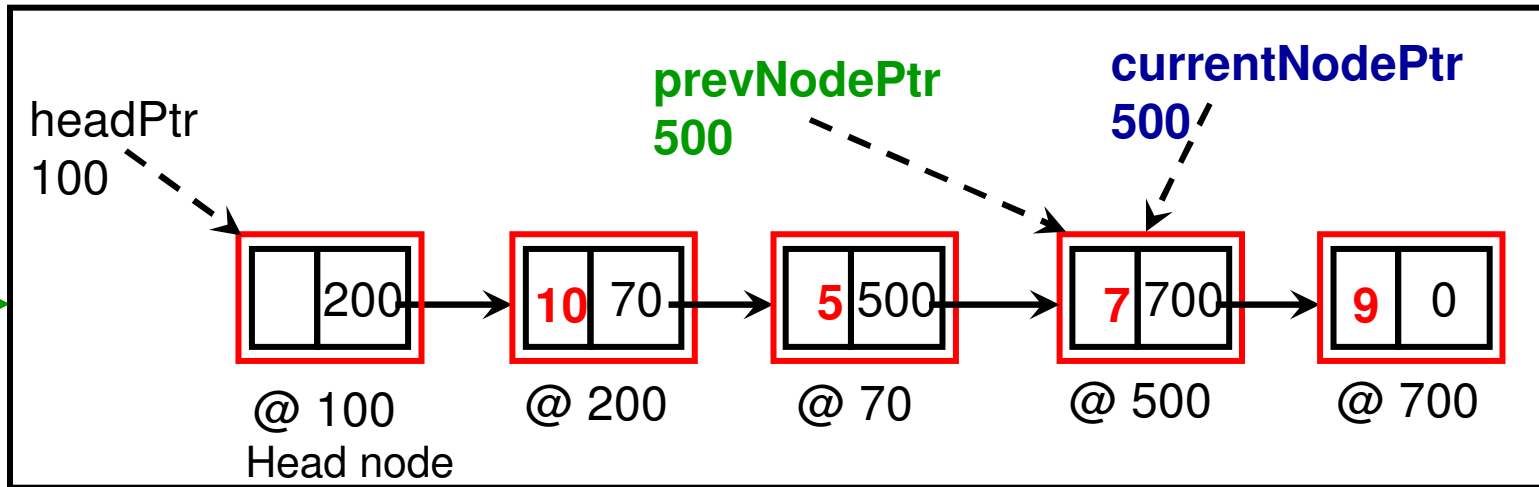
```
while (currentNodePtr != 0){
        prevNodePtr = currentNodePtr;
        currentNodePtr = currentNodePtr->getNextNodePtr();
}
```

**prevNodePtr**
**70**

**currentNodePtr**
**70**

headPtr
100

| | 200 | → | 10 | 70 | → | 5 | 500 | → | 7 | 700 | → | 9 | 0 |

@ 100
Head node

@ 200

@ 70

@ 500

@ 700

**prevNodePtr**
**70**

**currentNodePtr**
**500**

headPtr
100

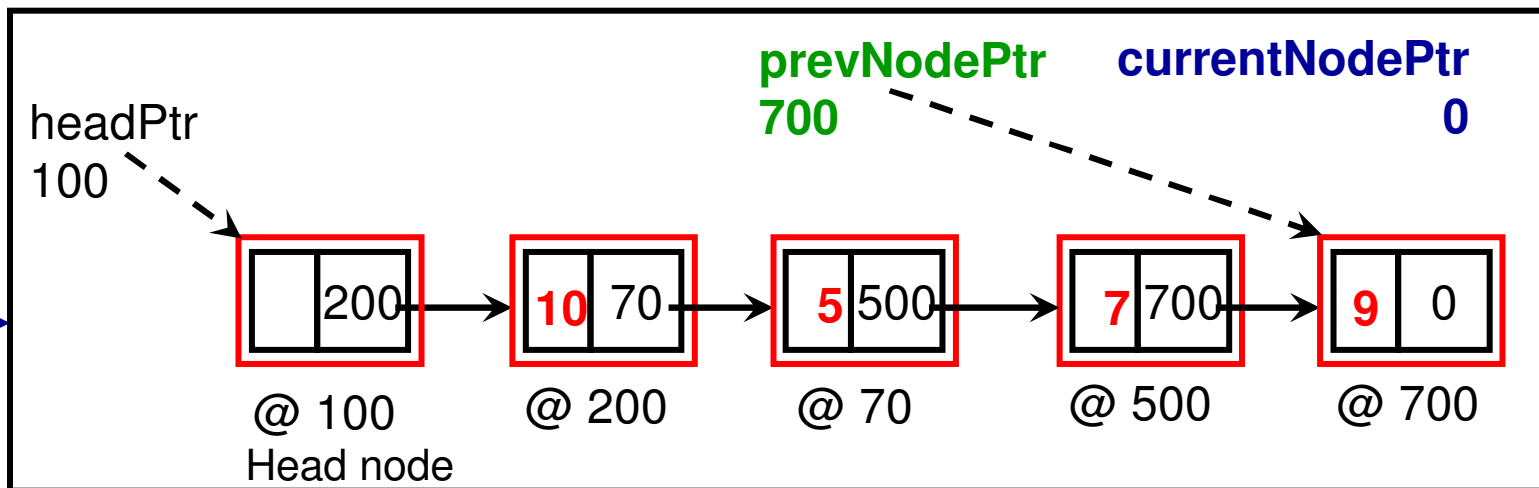| | 200 | → | 10 | 70 | → | 5 | 500 | → | 7 | 700 | → | 9 | 0 |

@ 100
Head node

@ 200

@ 70

@ 500

@ 700
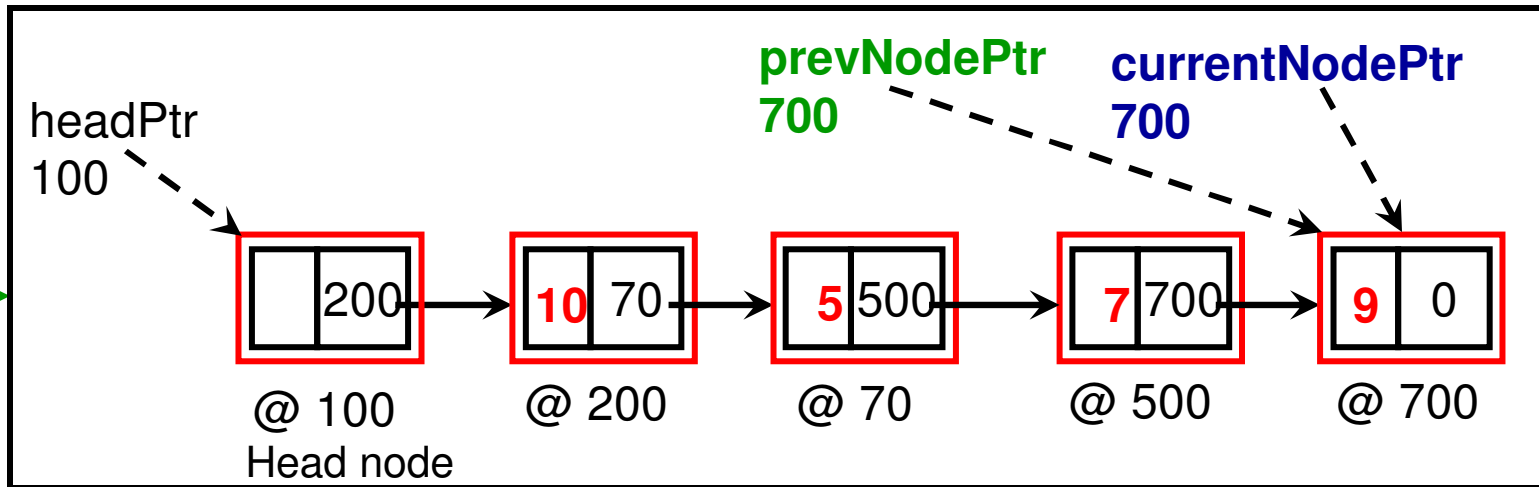
```
while (currentNodePtr != 0){
        prevNodePtr = currentNodePtr;
        currentNodePtr = currentNodePtr->getNextNodePtr();
}
```

```
while (currentNodePtr != 0){
        prevNodePtr = currentNodePtr;
        currentNodePtr = currentNodePtr->getNextNodePtr();
}
```

**prevNodePtr**    **currentNodePtr**
**700**            **700**

headPtr
100

| | 200 | → | 10 | 70 | → | 5 | 500 | → | 7 | 700 | → | 9 | 0 |

@ 100        @ 200        @ 70        @ 500        @ 700
Head node

**prevNodePtr**    **currentNodePtr**
**700**                        **0**

headPtr
100

| | 200 | → | 10 | 70 | → | 5 | 500 | → | 7 | 700 | → | 9 | 0 |

@ 100        @ 200        @ 70        @ 500        @ 700
Head node
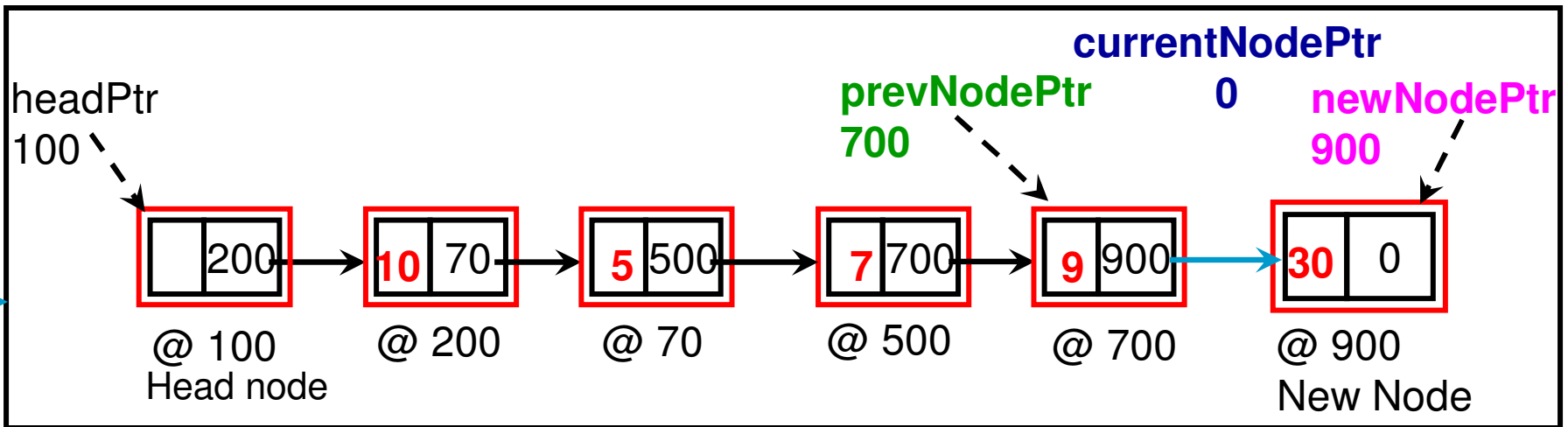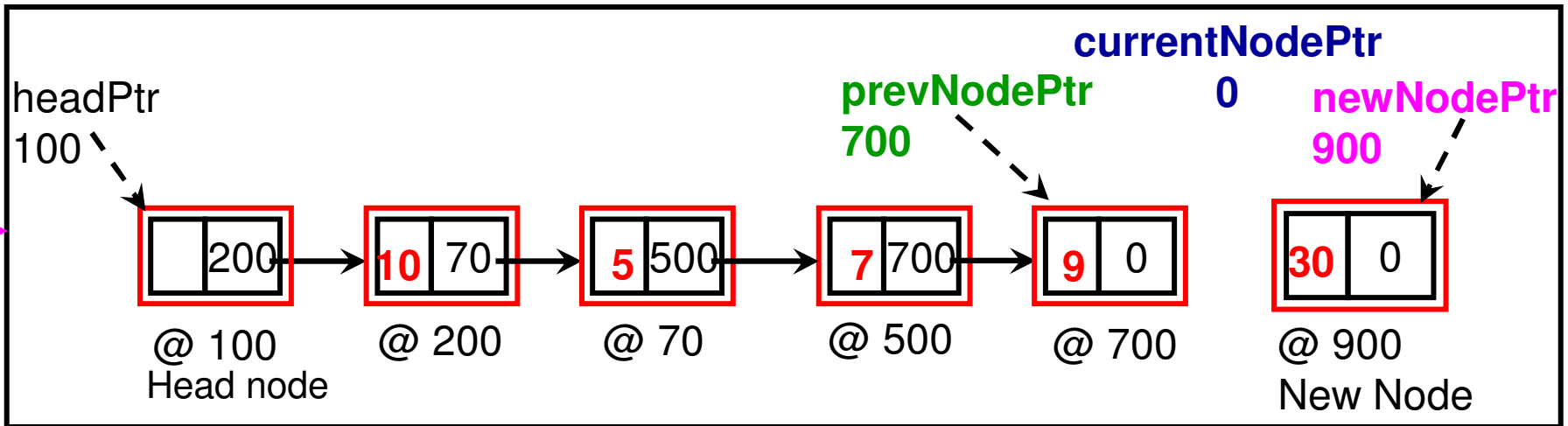
```
Node* newNodePtr = new Node();
newNodePtr->setData(data);
newNodePtr->setNextNodePtr(0);
prevNodePtr->setNextNodePtr(newNodePtr);
```

Let '30' be the data to be inserted at the end of the Linked List

**currentNodePtr**

headPtr
100

**prevNodePtr**
**700**

**0**

**newNodePtr**
**900**

| | 200 | 10 | 70 | 5 | 500 | 7 | 700 | 9 | 0 | 30 | 0 |

@ 100
Head node

@ 200

@ 70

@ 500

@ 700

@ 900
New Node

**currentNodePtr**

headPtr
100

**prevNodePtr**
**700**

**0**

**newNodePtr**
**900**

| | 200 | 10 | 70 | 5 | 500 | 7 | 700 | 9 | 900 | 30 | 0 |

@ 100
Head node

@ 200

@ 70

@ 500

@ 700

@ 900
New Node

## Class List (C++)

```cpp
void insertAtIndex(int insertIndex, int data){

    Node* currentNodePtr = headPtr->getNextNodePtr();
    Node* prevNodePtr = headPtr;

    int index = 0;

    while (currentNodePtr != 0){

        if (index == insertIndex)
            break;

        prevNodePtr = currentNodePtr;
        currentNodePtr = currentNodePtr->getNextNodePtr();
        index++;
    }

    Node* newNodePtr = new Node();
    newNodePtr->setData(data);
    newNodePtr->setNextNodePtr(currentNodePtr);
    prevNodePtr->setNextNodePtr(newNodePtr);

}
```
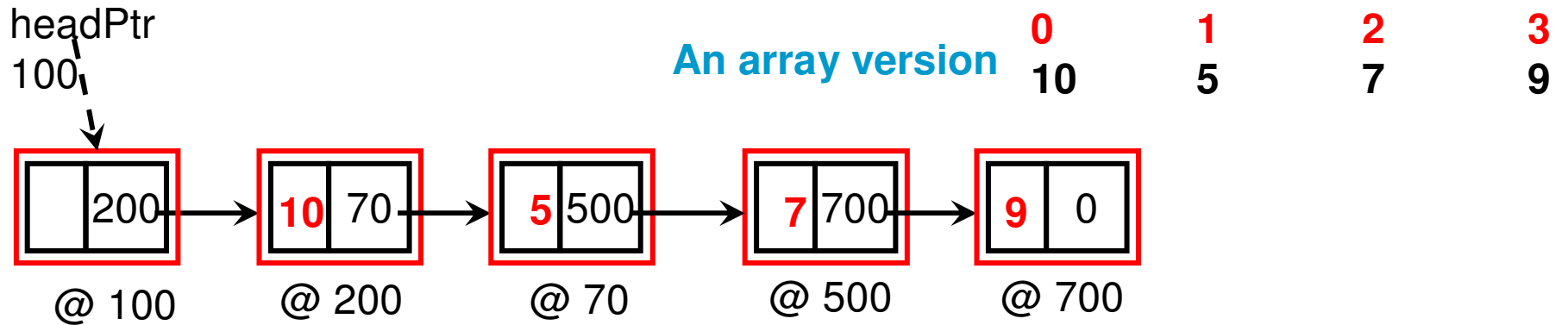
**index refers to the node pointed by currentNodePtr at any time**

**During the beginning and end of the while loop, the value for 'index' corresponds to the Position of the currentNode ptr and prevNode ptr corresponds to index-1.**

**If index equals insertIndex, we break from the while loop and insert the new node at the index in between prevNode and currentNode.**
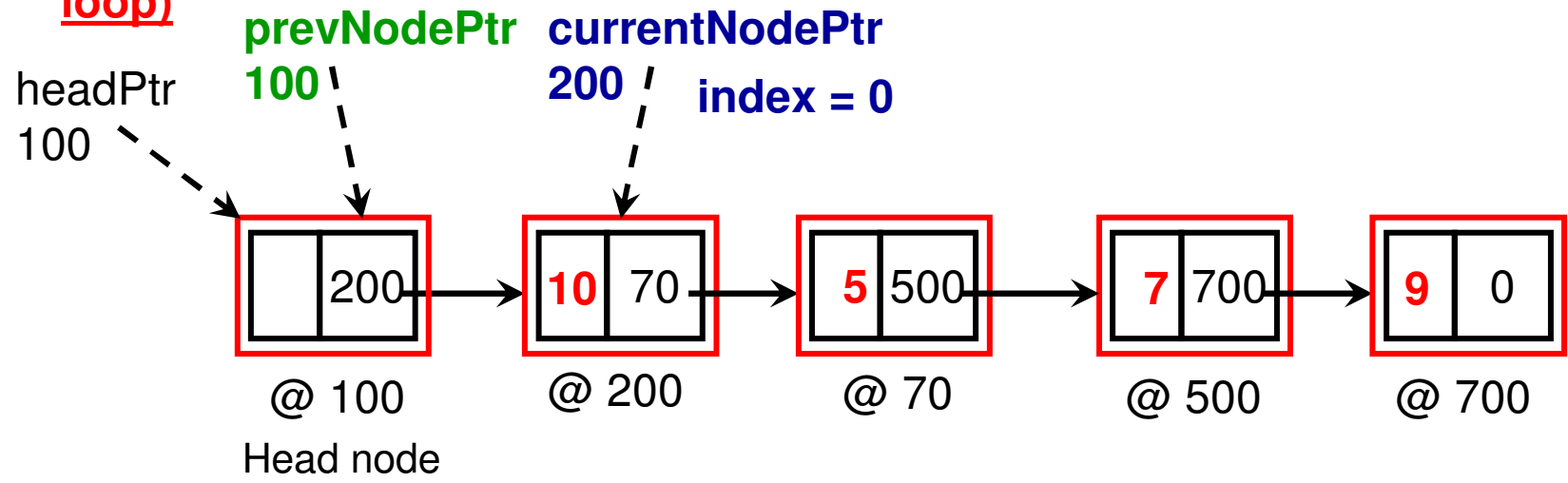
# Example: Insertion at *insertIndex* = 2 (1)

Let the List be **10  5  7  9** and let us say we want to insert element '30' at *insertIndex* = 2

headPtr
100

**An array version**

|  | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
|  | **10** | **5** | **7** | **9** |

| | 200 | → | **10** | 70 | → | **5** | 500 | → | **7** | 700 | → | **9** | 0 |

@ 100        @ 200        @ 70        @ 500        @ 700

Head node

```
Node* currentNodePtr = headPtr->getNextNodePtr();
Node* prevNodePtr = headPtr;
int index = 0;
```

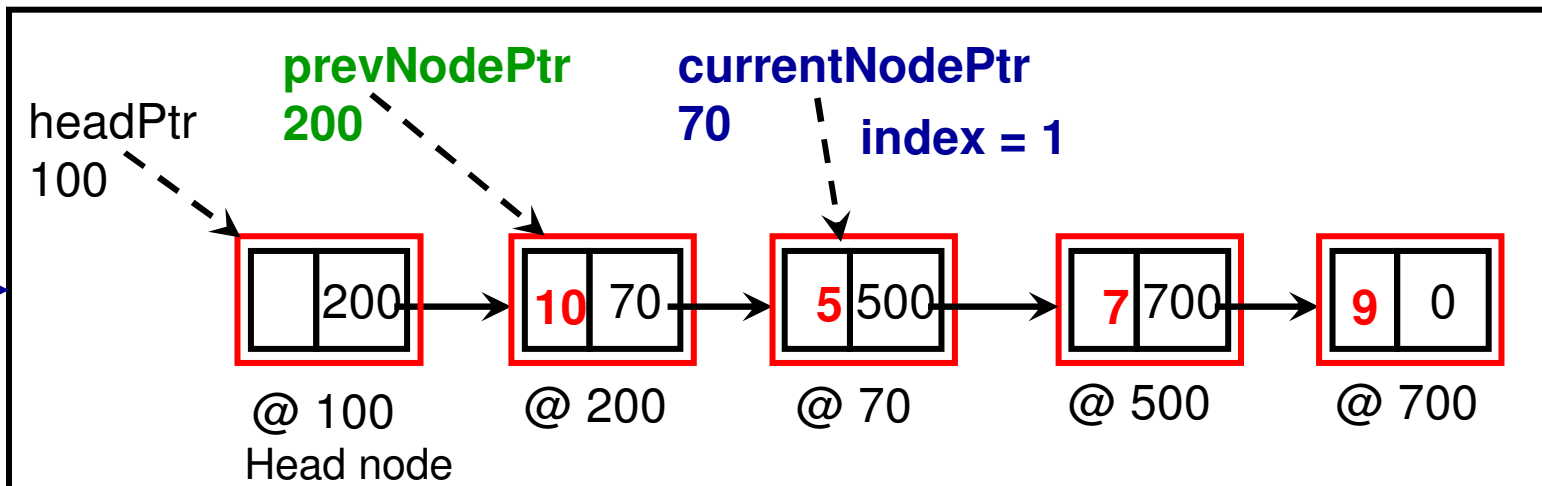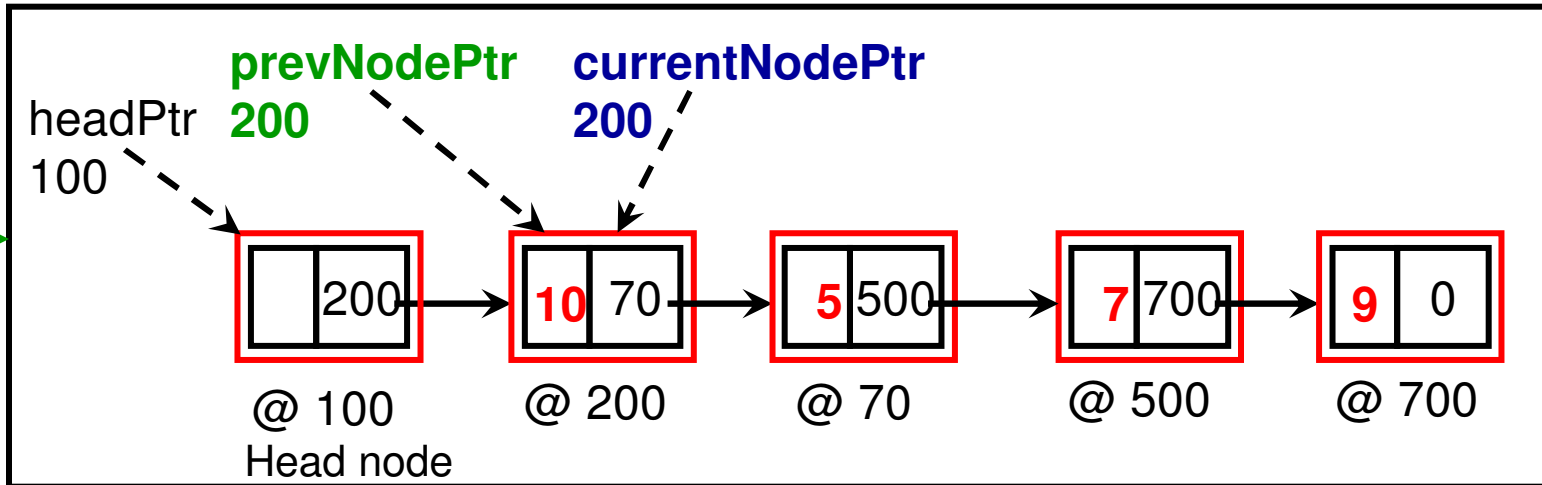**Initialization of prevNodePtr and currentNodePtr (before the while loop)**

**prevNodePtr**
**100**

**currentNodePtr**
**200**

**index = 0**

headPtr
100

| | 200 | → | **10** | 70 | → | **5** | 500 | → | **7** | 700 | → | **9** | 0 |

@ 100        @ 200        @ 70        @ 500        @ 700

Head node

*insertIndex = 2*

```
prevNodePtr = currentNodePtr;
currentNodePtr = currentNodePtr->getNextNodePtr();
index++;
```

**prevNodePtr**   **currentNodePtr**
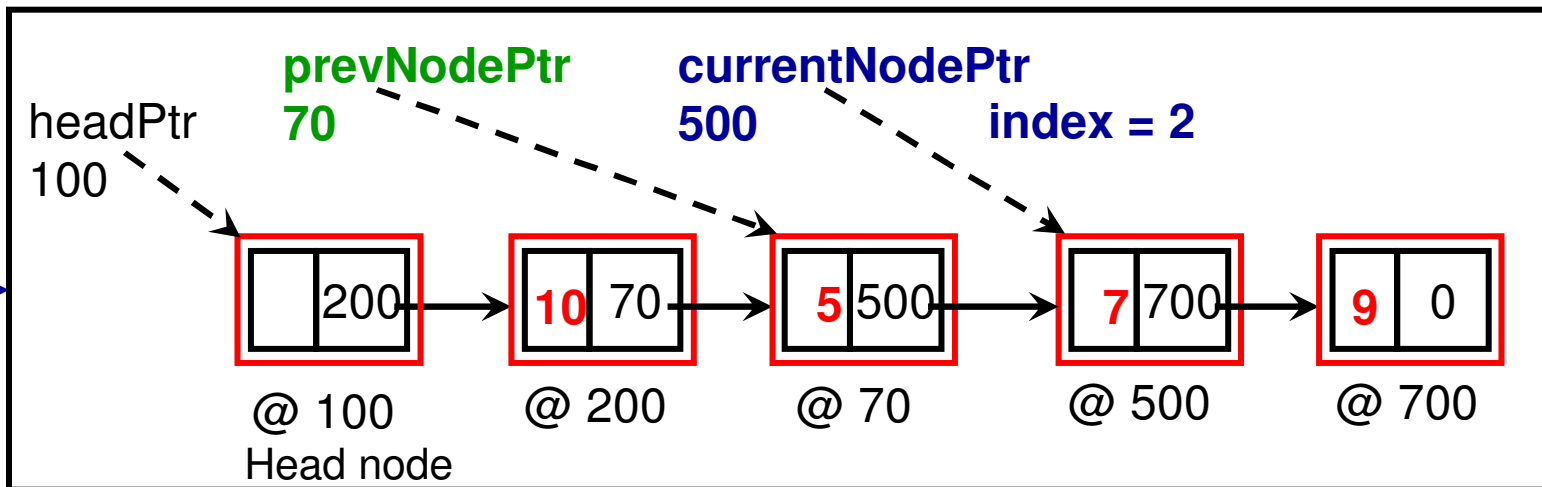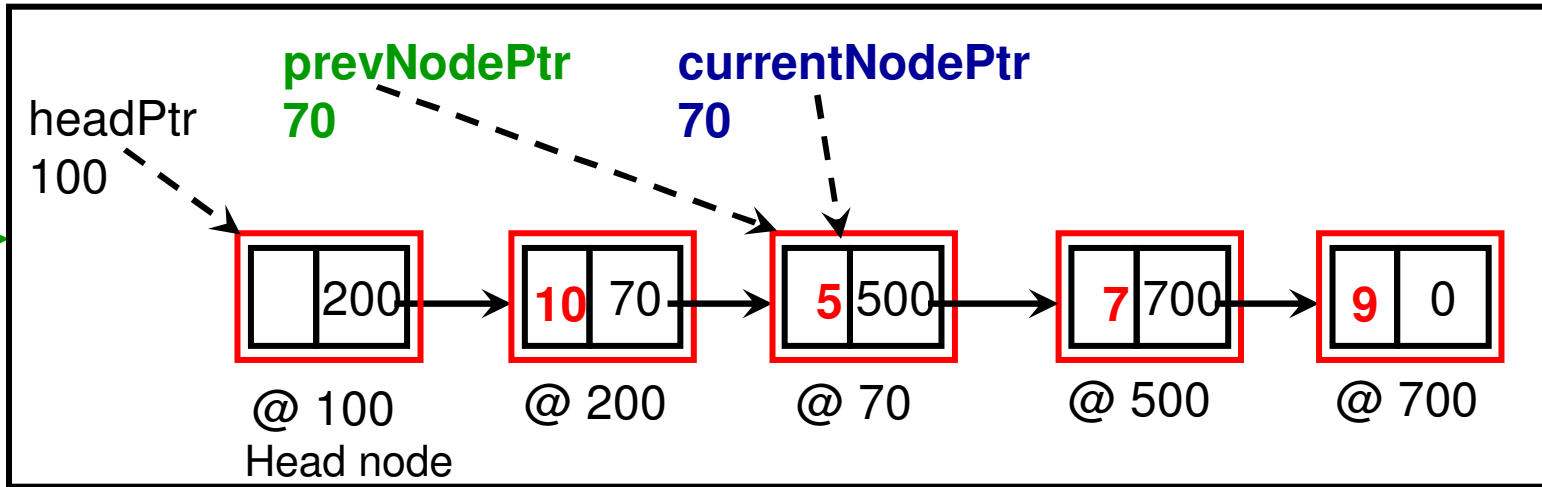**200**            **200**

headPtr
100

| | 200 | → | 10 | 70 | → | 5 | 500 | → | 7 | 700 | → | 9 | 0 |

@ 100        @ 200        @ 70        @ 500        @ 700
Head node

**prevNodePtr**        **currentNodePtr**
**200**                 **70**     **index = 1**

headPtr
100

| | 200 | → | 10 | 70 | → | 5 | 500 | → | 7 | 700 | → | 9 | 0 |

@ 100        @ 200        @ 70        @ 500        @ 700
Head node

```
prevNodePtr = currentNodePtr;
currentNodePtr = currentNodePtr->getNextNodePtr();
index++;
```

**prevNodePtr**     **currentNodePtr**

**prevNodePtr 70**

**currentNodePtr 70**

headPtr
100

| | 200 | → | 10 | 70 | → | 5 | 500 | → | 7 | 700 | → | 9 | 0 |

@ 100    @ 200    @ 70    @ 500    @ 700
Head node

**prevNodePtr 70**

**currentNodePtr 500**    **index = 2**

headPtr
100

| | 200 | → | 10 | 70 | → | 5 | 500 | → | 7 | 700 | → | 9 | 0 |

@ 100    @ 200    @ 70    @ 500    @ 700
Head node

**index = 2**          *insertIndex = 2*

**prevNodePtr**
**70**          **currentNodePtr**
**500**

headPtr
100

| | 200 | → | **10** | 70 | → | **5** | 500 | → | **7** | 700 | → | **9** | 0 |

@ 100          @ 200          @ 70          @ 500          @ 700
Head node

---

**prevNodePtr**
**70**          **currentNodePtr**
**500**

headPtr
100

**After breaking from**
**the while loop**
**Linking of the**
**newNode**

| | 200 | → | **10** | 70 | → | **5** | 900 | | **7** | 700 | → | **9** | 0 |

@ 100          @ 200          @ 70          @ 500          @ 700
Head node

| **30** | 500 |
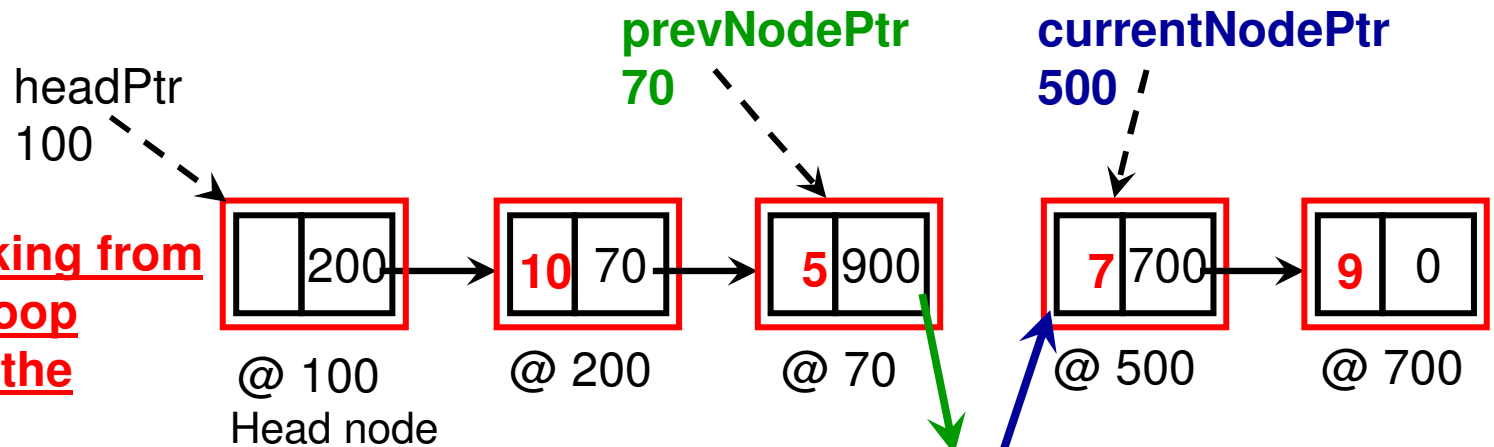
**newNode->setNextNodePtr(currentNode);**
**prevNode->setNextNodePtr(newNode);**

**newNodePtr**
**900**

@ 900
New Node

## Class List (C++)

```cpp
int read(int readIndex){
    Node* currentNodePtr = headPtr->getNextNodePtr();
    Node* prevNodePtr = headPtr;
    int index = 0;

    while (currentNodePtr != 0){

            if (index == readIndex)
                    return currentNodePtr->getData();

            prevNodePtr = currentNodePtr;
            currentNodePtr = currentNodePtr->getNextNodePtr();


            index++;

    }
    return -1; // an invalid value indicating index is out of range
}
```

**The 'index' value corresponds to the Position of the currentNode ptr and index-1 corresponds to prevNode ptr**

## Class List (C++)

```cpp
void modifyElement(int modifyIndex, int data){
        Node* currentNodePtr = headPtr->getNextNodePtr();
        Node* prevNodePtr = headPtr;
        int index = 0;

        while (currentNodePtr != 0){

                if (index == modifyIndex){
                        currentNodePtr->setData(data);
                        return;
                }

                prevNodePtr = currentNodePtr;
                currentNodePtr = currentNodePtr->getNextNodePtr();

                index++;
        }
}
```
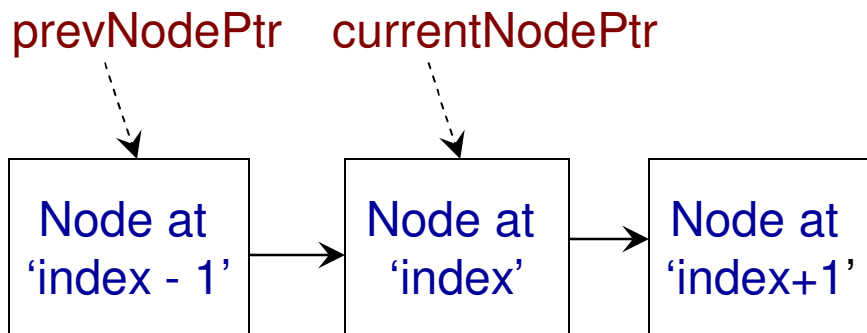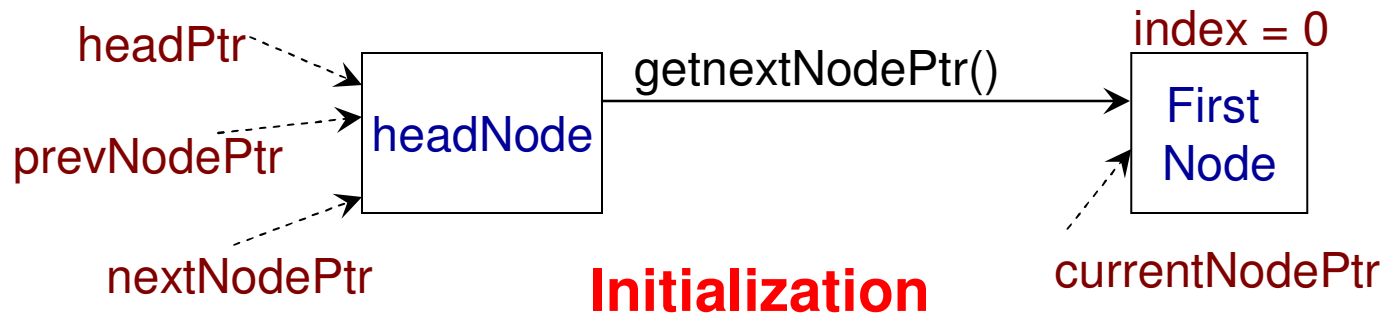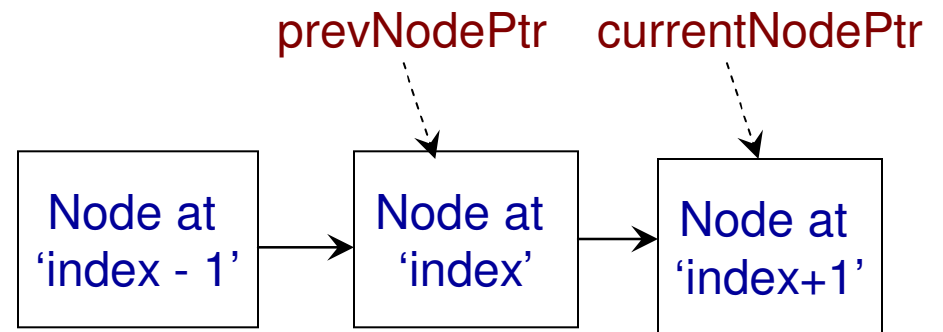
# Delete (deleteIndex) Function



headPtr

prevNodePtr

nextNodePtr

headNode

getnextNodePtr()

index = 0

First Node

currentNodePtr

**Initialization**

prevNodePtr    currentNodePtr

Node at 'index - 1' → Node at 'index' → Node at 'index+1'

**At the beginning of an iteration inside the 'while' loop**

prevNodePtr    currentNodePtr
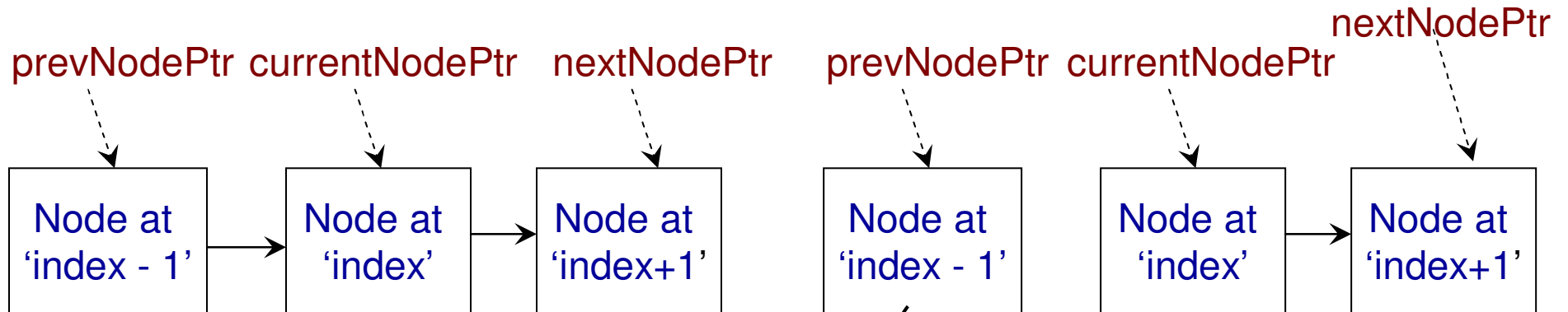
Node at 'index - 1' → Node at 'index' → Node at 'index+1'

**At the end of an iteration inside the 'while' loop**

**When index != deleteIndex**

# Delete (deleteIndex) Function

**When index == deleteIndex**

nextNodePtr

prevNodePtr   currentNodePtr          nextNodePtr          prevNodePtr   currentNodePtr

| Node at 'index - 1' | → | Node at 'index' | → | Node at 'index+1' |

**Inside the 'while' loop**

| Node at 'index - 1' | | Node at 'index' | → | Node at 'index+1' |

**Outside the 'while' loop**

**currentNode at index = deleteIndex
is disconnected from the Linked List**

## Class List (C++)

```cpp
void deleteElement(int deleteIndex){
    Node* currentNodePtr = headPtr->getNextNodePtr();
    Node* prevNodePtr = headPtr;
    Node* nextNodePtr = headPtr;
    int index = 0;

    while (currentNodePtr != 0){

        if (index == deleteIndex){
            nextNodePtr = currentNodePtr->getNextNodePtr();
            break;
        }

        prevNodePtr = currentNodePtr;
        currentNodePtr = currentNodePtr->getNextNodePtr();


        index++;
    }

    prevNodePtr->setNextNodePtr(nextNodePtr);

}
```
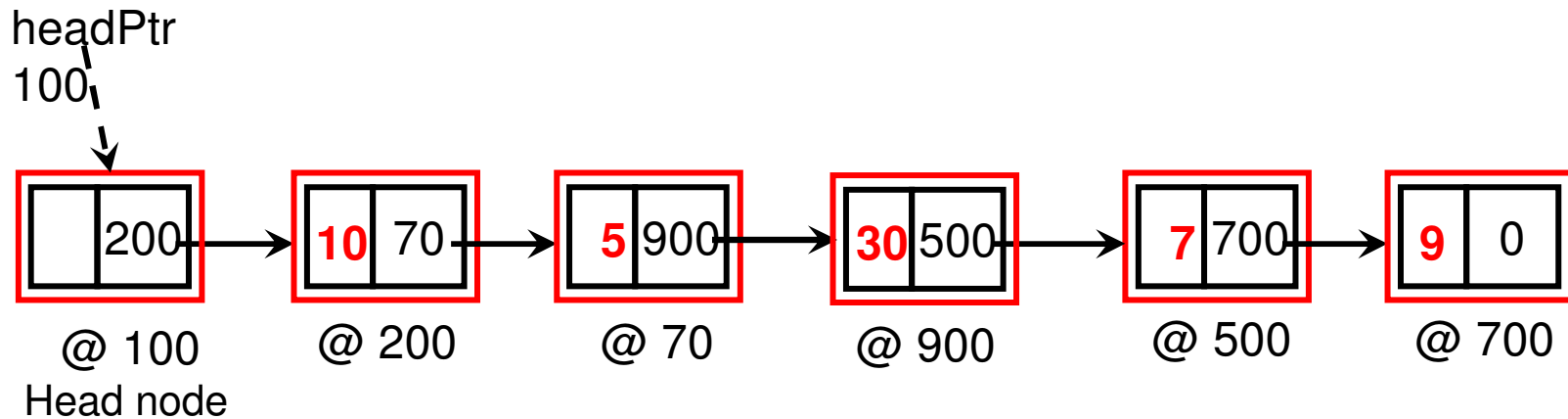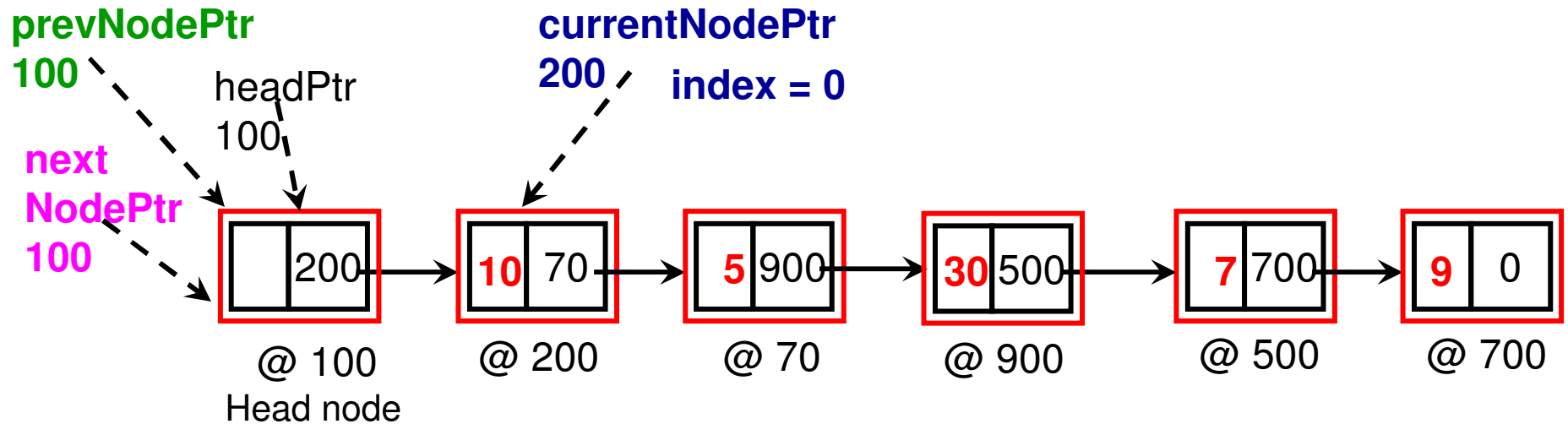
**The next node for 'prevNode' ptr is now 'next node' and not 'current node'**

# Example: Deletion at *deleteIndex = 2*

Let the List be **10  5  30  7  9** and now we want to delete '30' at *deleteIndex* = 2
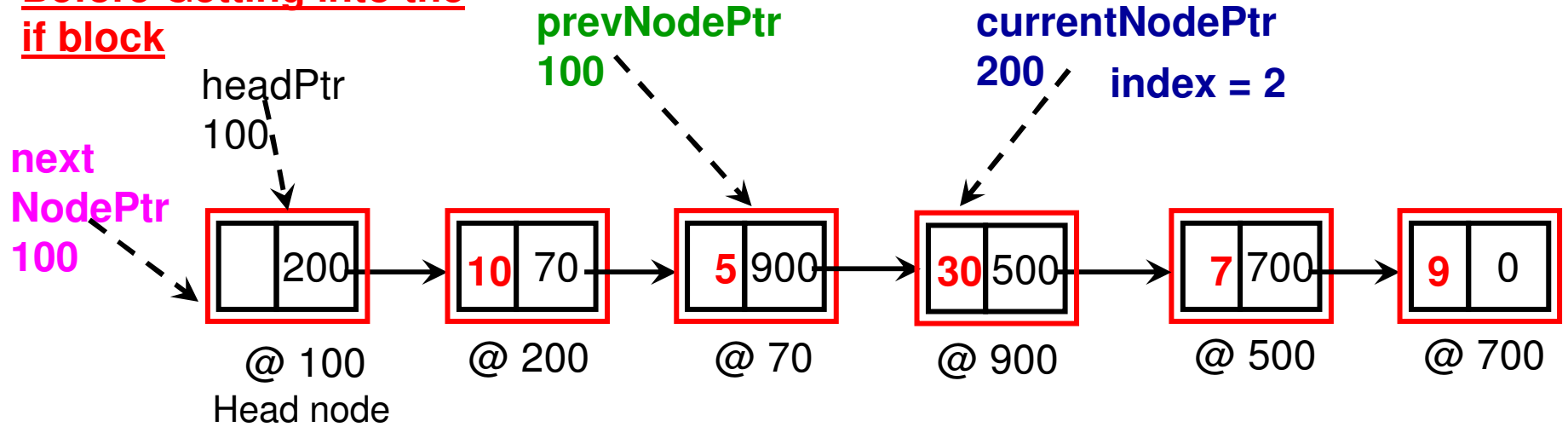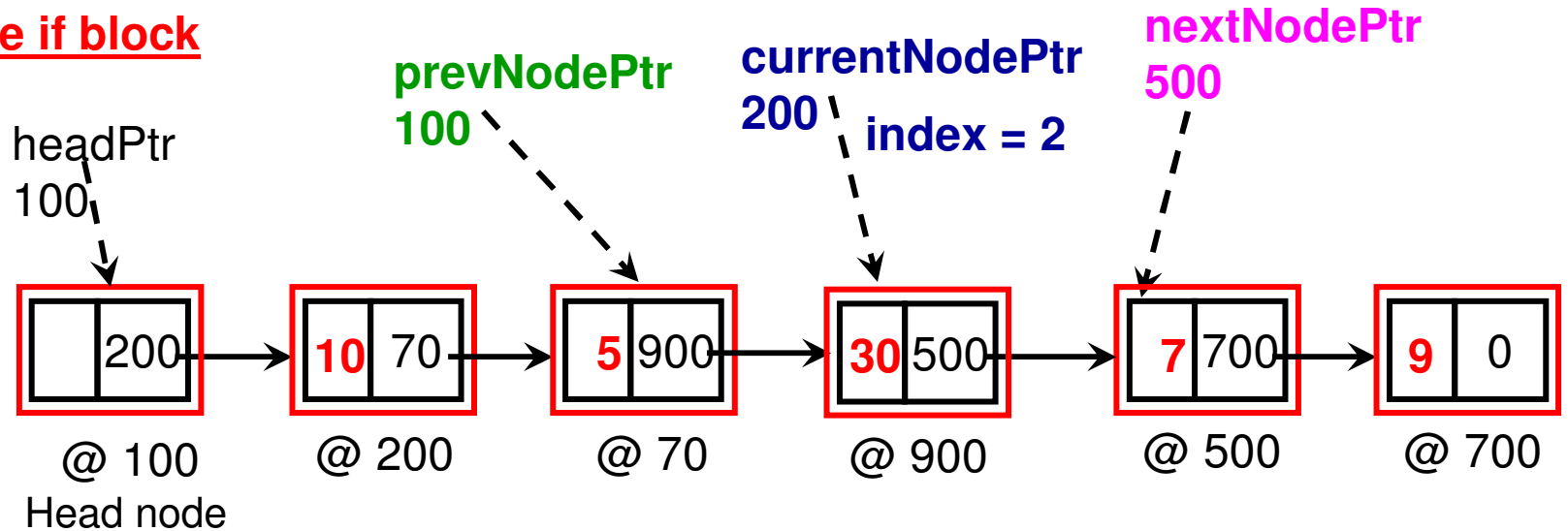
headPtr
100

| | 200 | → | **10** | 70 | → | **5** | 900 | → | **30** | 500 | → | **7** | 700 | → | **9** | 0 |

@ 100        @ 200        @ 70        @ 900        @ 500        @ 700
Head node

## Initialization of the pointers

**prevNodePtr**
**100**

headPtr
100

**currentNodePtr**
**200**        **index = 0**

**next**
**NodePtr**
**100**

| | 200 | → | **10** | 70 | → | **5** | 900 | → | **30** | 500 | → | **7** | 700 | → | **9** | 0 |

@ 100        @ 200        @ 70        @ 900        @ 500        @ 700
Head node

# Example: Deletion at *deleteIndex = 2*

prevNodePtr
100

currentNodePtr
200    index = 2

headPtr
100

next
NodePtr
100

| | 200 | → | 10 | 70 | → | 5 | 900 | → | 30 | 500 | → | 7 | 700 | → | 9 | 0 |

@ 100        @ 200      @ 70       @ 900      @ 500      @ 700
Head node

---

**Inside the if block**

nextNodePtr
500

prevNodePtr
100

currentNodePtr
200    index = 2

headPtr
100

| | 200 | → | 10 | 70 | → | 5 | 900 | → | 30 | 500 | → | 7 | 700 | → | 9 | 0 |

@ 100        @ 200      @ 70       @ 900      @ 500      @ 700
Head node

# Example: Deletion at *deleteIndex = 2*

nextNodePtr
500

prevNodePtr
100

currentNodePtr
200

index = 2

headPtr
100

| | 200 | → | 10 | 70 | → | 5 | 500 | | 30 | 500 | → | 7 | 700 | → | 9 | 0 |

@ 100          @ 200          @ 70          @ 900          @ 500          @ 700
Head node
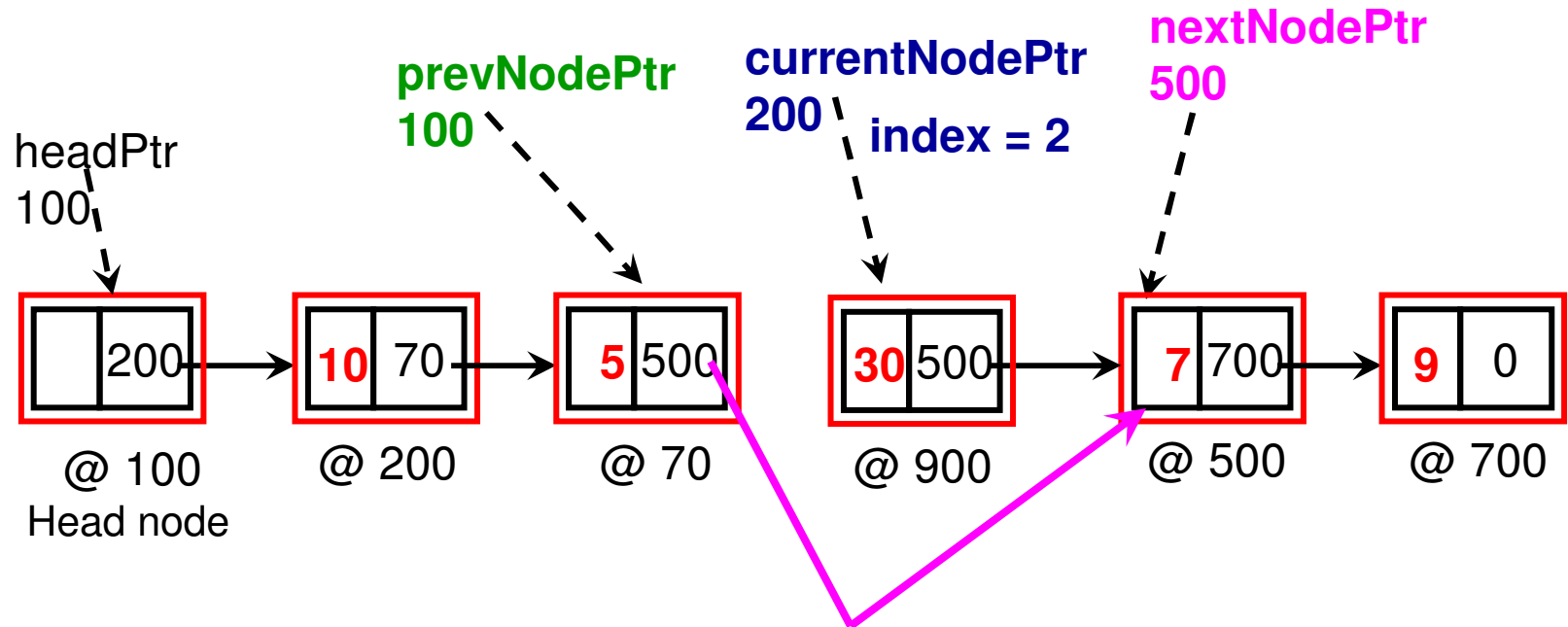
**Delinking of Node at index = 2 (element '30' from the Linked List)**

# Linked List vs. Arrays: Memory Usage

|  | Data size | Next Node Ptr | Prev Node Ptr | Node Size |
|---|---|---|---|---|
| Singly Linked List | 4 (int) | 4 | N/A | 8 bytes |
| Singly Linked List | 32 | 4 | N/A | 36 bytes |
| Doubly Linked List | 4 (int) | 4 | 4 | 12 bytes |
| Doubly Linked List | 32 | 4 | 4 | 40 bytes |

An array is usually considered to take space that is twice the number of elements in it. Still, it looks like the Linked Lists will take a larger memory compared to an array. But, it is not always the case.

Consider a scenario wherein 64,000 objects (each of size 32 bytes) are to be stored in a List.

If we were to stored the objects in an array, there would need to be space for 128,000 objects. Hence, a dynamic array-based implementation will now hold up 128,000 * 32 bytes = 40,96,000 bytes in memory.

A singly linked list based implementation will hold (64,000 + 1 head node) * 36 bytes = 23,04,036 bytes in memory.

A doubly linked list based implementation will hold (64,000 + 1 head node + 1 tail node)* 40 bytes = 25,60,080 bytes in memory.

# Linked List vs. Arrays: Memory Usage

On the other hand, Consider a scenario wherein 4,000 integers (each integer is 4 bytes) are to be stored in a List.

An array-based implementation will now hold 8,000 * 4 = 32,000 bytes in memory.

A singly linked list-based implementation will now hold (4,000 + 1 head node) * 8 = 32,008 bytes in memory.

A doubly linked list-based implementation will now hold (4,000 + 1 head node + 1 tail node) * 12 = 48,024 bytes in memory.

# Linked List vs. Arrays: Time Complexity

| | Array | Singly Linked List | Doubly Linked List |
|---|---|---|---|
| Read/Modify | Θ(1) | O(n) | O(n) |
| Insert | O(n) | O(n) | O(n) |
| Delete | O(n) | O(n) | O(n) |
| isEmpty | Θ(1) | Θ(1) | Θ(1) |
| Count | Θ(1) | O(n) | O(n) |

We typically use arrays if there are more frequent read/modify operations compared to Insert/Delete

We typically use Linked Lists if there are more frequent insert/delete operations compared to read/modify (remember: arrays come with the overhead of creating a new block of memory, if needed, and copying the elements to the new block)

**Note: With arrays, Insert operations are more time consuming if need to be done at the smaller indices. With singly linked lists, insert operations are more time consuming if done towards the end of the list. A doubly linked list could be traversed either from the head or the tail, and hence if a priori information is know about the sequence of elements in the list, traversal could be initiated from the head or tail, and the traversal time could be lower than a singly linked list. Still O(n) time though!**

# Sorting Algorithm: Selection Sort

- Given an array A[0…n-1], we proceed for a total of n-1 iterations)
- In iteration i ($0 \leq i < n-1$), we assume A[i] is the minimum element and seek to find whether there exists an element at index i+1…n-1 so that we can swap that element with A[i], if such an element exists.

|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Given Array** | 5 | 6 | 5 | 4 | 3 | 10 | 9 | 1 | 7 | 8 |

|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Iteration 0** | 5 | 6 | 5 | 4 | 3 | 10 | 9 | 1 | 7 | 8 |

|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Iteration 0 (After)** | 1 | 6 | 5 | 4 | 3 | 10 | 9 | 5 | 7 | 8 |

|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Iteration 1** | 1 | 6 | 5 | 4 | 3 | 10 | 9 | 5 | 7 | 8 |

|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Iteration 1 (After)** | 1 | 3 | 5 | 4 | 6 | 10 | 9 | 5 | 7 | 8 |

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Iteration 2** | 1 | 3 | 5 | 4 | 6 | 10 | 9 | 5 | 7 | 8 |

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Iteration 2 (After)** | 1 | 3 | 4 | 5 | 6 | 10 | 9 | 5 | 7 | 8 |

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Iteration 3** | 1 | 3 | 4 | 5 | 6 | 10 | 9 | 5 | 7 | 8 |

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Iteration 3 (After)** | 1 | 3 | 4 | 5 | 6 | 10 | 9 | 5 | 7 | 8 |

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Iteration 4** | 1 | 3 | 4 | 5 | 6 | 10 | 9 | 5 | 7 | 8 |

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Iteration 4 (After)** | 1 | 3 | 4 | 5 | 5 | 10 | 9 | 6 | 7 | 8 |

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| Iteration 8 | 1 | 3 | 4 | 5 | 5 | 6 | 7 | 8 | 9 | 10 |

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| Iteration 8 (After) | 1 | 3 | 4 | 5 | 5 | 6 | 7 | 8 | 9 | 10 |

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| Final Sorted Array | 1 | 3 | 4 | 5 | 5 | 6 | 7 | 8 | 9 | 10 |

# Selection Sort

## ALGORITHM

//Input: An array $A[0..n-1]$ of orderable elements
//Output: Array $A[0..n-1]$ sorted in nondecreasing order
**for** $i \leftarrow 0$ **to** $n - 2$ **do**
    $min \leftarrow i$
    **for** $j \leftarrow i + 1$ **to** $n - 1$ **do**
        **if** $A[j] < A[min]$  $min \leftarrow j$
    swap $A[i]$ and $A[min]$

**# Comparisons**
$(n-1) + (n-2) + \ldots + 1 = n(n-1)/2 = \Theta(n^2)$

There is no best or worst case. In the ith Iteration, we have to find if there exists any element that is less than the element at index i.

**Code 7** Find the address of the node with the minimum data

```
Node* findMinimumDataNodeAddress(List list){

    Node* headPtr = list.getHeadPtr();
    Node* currentNodePtr = headPtr->getNextNodePtr();

    Node* minDataNodePtr;

    if (currentNodePtr != 0){
        minDataNodePtr = currentNodePtr;
        currentNodePtr = currentNodePtr->getNextNodePtr();
    }
    else{   // return '0' (null) if the list is empty
        return 0;
    }


    while (currentNodePtr != 0){

        if (minDataNodePtr->getData() > currentNodePtr->getData())
            minDataNodePtr = currentNodePtr;

        currentNodePtr = currentNodePtr->getNextNodePtr();

    }

    return minDataNodePtr;

}
```

/* Assigns the address of the first data node as the initial value of minDataNodePtr */

/* Inside this loop, minDataNodePtr will be set to the address of the node (if any exists) whose data is less than the data of the node whose address is stored in minDataNodePtr */

```
Node* MinDataNodePtr = findMinimumDataNodeAddress(integerList);
if (MinDataNodePtr != 0)
        cout << "Minimum data is: " << MinDataNodePtr->getData() << endl;
else
        cout << "The list is empty!!" << endl;
```

# Iterative Print

```cpp
void IterativePrint(){          Class List (C++)

    Node* currentNodePtr = headPtr->getNextNodePtr();

    while (currentNodePtr != 0){
            cout << currentNodePtr->getData() << " ";
            currentNodePtr = currentNodePtr->getNextNodePtr();
    }

    cout << endl;

}
```

# Recursion

- Recursion: A function calling itself.
- Recursions are represented using a recurrence relation (incl. a base or terminating condition)

- Example 1
- Factorial (n) = n * Factorial(n-1) for n > 0
- Factorial (n) = 1 for n = 0

```
Factorial(n)
  if (n == 0)
      return 1;
  else
      return n * Factorial(n-1)
```

**Memory Stack**

| |
|---|
| **Factorial (0) = 1** |
| **Factorial (1) = 1 * Factorial (0)** |
| **Factorial (2) = 2 * Factorial (1)** |
| **Factorial (3) = 3 * Factorial (2)** |
| **Factorial (4) = 4 * Factorial (3)** |
| **Factorial (5) = 5 * Factorial (4)** |

# Example (Code 4) to Illustrate Recursion and Random Number Generation

**C++**

```cpp
int main(){

    int arraySize;
    cout << "Enter an array size: ";
    cin >> arraySize;

    int maxValue;
    cout << "Enter the max. value of an element: ";
    cin >> maxValue;

    srand(time(NULL));

    int array[maxValue];

    for (int i = 0; i < arraySize; i++){
        array[i] = rand() % (1 + maxValue);
    }

    cout << "IterativePrint: ";
    IterativePrint(array, arraySize);

    cout << "RecursivePrint: " << endl;
    RecursivePrint(array, arraySize, 0);

    return 0;
}
```

**Initialize the random number generator with a seed that corresponds to the current system time**

**The random numbers are generated from 1 to maxValue**

**Headers to be included**

```cpp
#include <iostream>
#include <stdlib.h> // random number
#include <time.h> // for time
using namespace std;
```

**Code 4: C++**

```cpp
void IterativePrint(int* arrayPtr, int size){
    for (int index = 0; index < size; index++)
        cout << arrayPtr[index] << " ";

    cout << endl;
}
```

```cpp
void RecursivePrint(int* arrayPtr, int size, int printIndex){
    if (printIndex == size){
        cout << endl;
        return;
    }
    cout << arrayPtr[printIndex] << " ";      // Printing in the forward order

    RecursivePrint(arrayPtr, size, printIndex+1);

    cout << arrayPtr[printIndex] << " ";      // Printing in the reverse order
}
```

# Recursion

**Seq**

|  | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| **array** | 14 | 21 | 33 | 45 |

| if (printIndex == size){ | // printIndex = 4; size = 4 |
|---|---|
| **10**   cout << endl; | // 4 == 4 |
| **11**   return; | |
| } | |

| Seq | 2 | 4 | 6 | 8 |
|---|---|---|---|---|
|  | 14 | 21 | 33 | 45 |
|  | 45 | 33 | 21 | 14 |
| Seq | 12 | 13 | 14 | 15 |

**8** cout << arrayPtr[3] << " "; // printIndex = 3 ; size = 4
**9** RecursivePrint(arrayPtr, size = 4, printIndex + 1)
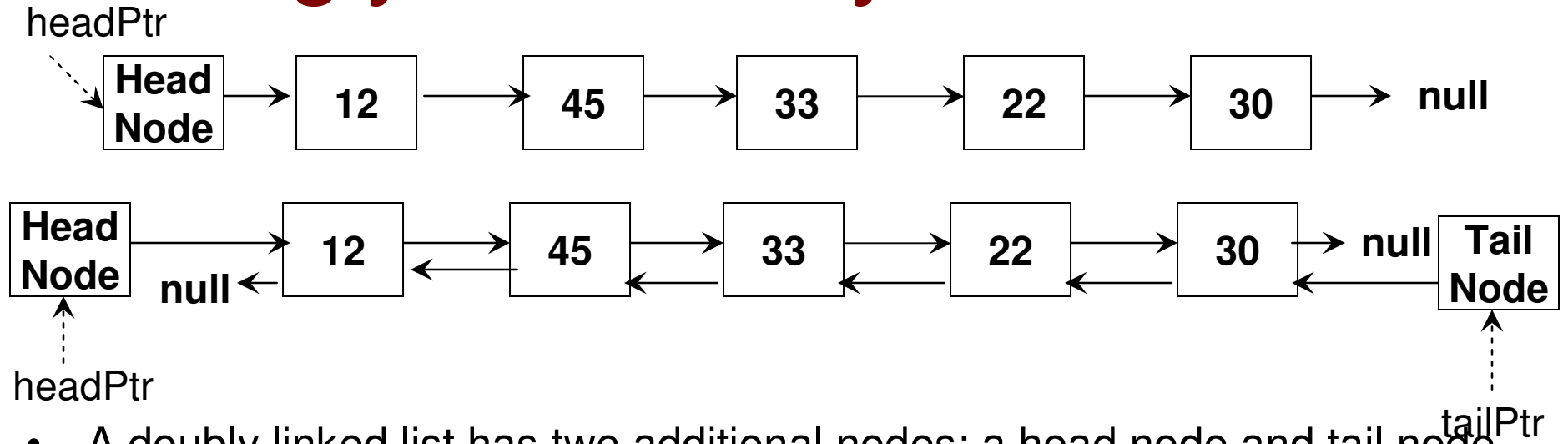**12** cout << arrayPtr[3] << " "; // printIndex = 3

**6** cout << arrayPtr[2] << " "; // printIndex = 2 ; size = 4
**7** RecursivePrint(arrayPtr, size = 4, printIndex + 1)
**13** cout << arrayPtr[2] << " "; // printIndex = 2

**4** cout << arrayPtr[1] << " "; // printIndex = 1; size = 4
**5** RecursivePrint(arrayPtr, size = 4, printIndex + 1)
**14** cout << arrayPtr[1] << " "; // printIndex = 1

**2** cout << arrayPtr[0] << " "; // printIndex = 0; size = 4
**3** RecursivePrint(arrayPtr, size = 4, printIndex + 1)
**15** cout << arrayPtr[0] << " "; // printIndex = 0

@main
**1** RecursivePrint(array, arraySize = 4, printIndex = 0)

# Singly vs. Doubly Linked List

headPtr

| Head Node | 12 | 45 | 33 | 22 | 30 | null |

| Head Node | 12 | 45 | 33 | 22 | 30 | null | Tail Node |

null ←

headPtr

tailPtr

- A doubly linked list has two additional nodes: a head node and tail node (a head ptr points to the head node whose next node is the first node in the list, and a tail ptr points to the tail node whose prev node is the last node in the list).
  - Note the next node for the last node in the list is null (so that the end of the list could be traced) as well as the prev node for the first node in the list is null (so that the beginning of the list could be traced).
- A doubly linked list could be traversed in either direction (from head to tail or from tail to head).
  - NextNodePtr values at the nodes are used to access in the forward direction (from head node to the last node)
  - PrevNodePtr values at the nodes are used to access in the reverse direction (from the tail node to the first node)

**Class Node**

**C++**

```cpp
private:
        int data;
        Node* nextNodePtr;
```

**Singly Linked List**

```cpp
public:
    Node(){}

    void setNextNodePtr(Node* nodePtr){
            nextNodePtr = nodePtr;
    }

    Node* getNextNodePtr(){
            return nextNodePtr;
    }
```
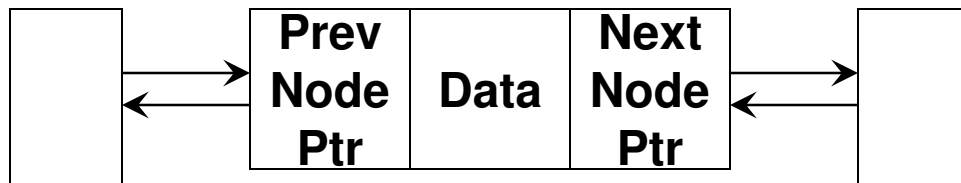
```cpp
private:
        int data;
        Node* nextNodePtr;
        Node* prevNodePtr;
```

**Doubly Linked List**

```cpp
public:
    Node(){}

    void setNextNodePtr(Node* nodePtr){
            nextNodePtr = nodePtr;
    }

    Node* getNextNodePtr(){
            return nextNodePtr;
    }
```
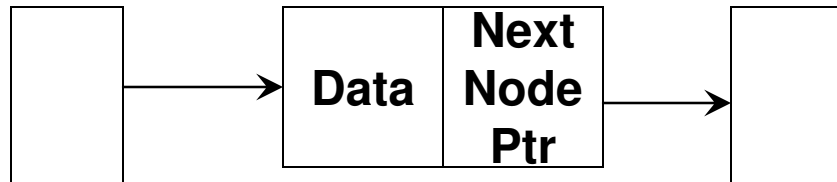
```cpp
void setPrevNodePtr(Node* nodePtr){
            prevNodePtr = nodePtr;
}

Node* getPrevNodePtr(){
        return prevNodePtr;
}
```

| | Data | Next Node Ptr | |
|---|---|---|---|

| | Prev Node Ptr | Data | Next Node Ptr | |
|---|---|---|---|---|

**Singly Linked List**

```cpp
private:
    Node *headPtr;

public:
    List( ){
        headPtr = new Node();
        headPtr->setNextNodePtr(0);
    }

    Node* getHeadPtr(){
        return headPtr;
    }
```

**Doubly Linked List**

```cpp
private:
    Node *headPtr;
    Node* tailPtr;

public:
    List(){
        headPtr = new Node();
        tailPtr = new Node();
        headPtr->setNextNodePtr(0);
        tailPtr->setPrevNodePtr(0);
    }

    Node* getHeadPtr(){
        return headPtr;
    }

    Node* getTailPtr(){
        return tailPtr;
    }
```